Problem sheet 11
March 22nd, 2005

MT290 Complex variable

Ex. 1
Evaluate the integral $\int_{|z|=1} \frac{e^{-z}}{z^2}\, dz$ (taken anticlockwise)
a) by means of Cauchy's integral formulae
b) by evaluating the residue
Do the same for $\int_{|z|=1} \frac{\sin(2z)}{z^2}\, dz$
$\int_{|z|=1} \frac{\sin(z)}{z^5}\, dz$
$\int_{|z|=1} \frac{\cosh z}{z^2}\, dz$
$\int_{|z|=1} \frac{\cosh z}{z^3}\, dz$.

Ex. 2

Note: some of the residues have been calculated on the previous example sheet!
Calculate the following two integrals:

$$\int_{-\infty}^{\infty} \frac{\cos x}{(x^2+1)(x^2+4)}\, dx, \qquad \int_{-\infty}^{\infty} \frac{\sin x}{(x^2+1)(x^2+4)}\, dx.$$

Ex. 3
Calculate the integral

$$\int_C \frac{1}{e^z+1}\, dz$$

around the circle $C : |z| = 3$ and around the circle $C : |z| = 4$.

Ex. 4
Prove (using Complex variable techniques) that

$$\int_{-\infty}^{\infty} \frac{x^2}{(x^2+1)(x^2+4)}\, dx = \frac{\pi}{3}.$$

Clearly state those results from the lecture that you use.

I hope you find some time for preparing your exams. I intend to give a general revision class about a week before the exam. Based on your questions!
I updated the webpage. All problem sheets and the mathematica notebook (on the Fundamental Theorem) are online.
http://www.ma.rhul.ac.uk/∼elsholtz/WWW/lectures/0405mt290/lecture.html
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