Ex. 1
(Not to be handed in) Work through pages 74-78 of Raymond Hill's book. In particular watch out for the advantages of incomplete decoding.

Ex. 2
Show that the decimal code

$$\left\{(x_1, x_2, \ldots, x_{10}) \in \mathbb{Z}_{10}^{10} | \sum_{i=1}^{10} x_i \equiv 0(\text{mod} 10), \sum_{i=1}^{10} i x_i \equiv 0(\text{mod} 10)\right\}$$

is not a single-error-correcting code. (Here $\mathbb{Z}_{10}$ denotes the set of integers mod 10.

Ex. 3
(see problem sheet 6) Let C be the ternary linear code with generator matrix

$$\begin{bmatrix} 1 & 1 & 1 & 0 \\ 2 & 0 & 1 & 1 \end{bmatrix}.$$

a) Find a generator matrix for C in standard form
b) Find a parity check matrix for C in standard form.
c) Use syndrome decoding to decode the received vectors 2121, 1201 and 2222.

Ex. 4
Suppose a certain binary channel accepts word of length 7 and that the only kind of error vector ever observed is one of the eight vectors

0000000, 0000001, 0000011, 0000111, 0001111, 0011111, 0111111, 111111.

Design a binary linear $[7, k]$-code which will correct all such errors with as large a rate as possible.

Ex. 5
Let C be the code generated by

$$G = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 \end{bmatrix}.$$

A new code C_2 is formed by adding a parity check.

a) Write down the generator matrix for C_2 in standard form.

b) Determine the parameters $[n, k, d]$ and M.

c) Write down its parity check matrix for C_2.

d) Show that C_2 can be used to simultaneously correct one error and detect two errors.

e) Make a lookup table to enable syndrome decoding to use it in the way suggested in *d*.

f) Decode the received vectors $0110110, 1111010$ and 1110111.

Do not write down the standard array or the codewords.