Problem sheet 10 2006

Ex. 1

Show that the Latin squares L_1 and L_3 of order 4 are not orthogonal. Construct two orthogonal Latin Squares of order 4 as follows:

Construct the field of 4 elements by taking all polynomials with coefficients 0 and 1, and reduce modulo $x^2 + x + 1$.

Show that the 4 elements are: 0, 1, x, x + 1.

Write down the addition and multiplication table of this field. From this find two MOLS of order 4. Bring them into standard form.

Ex. 2

- Show that $f(x) = x^3 + x + 1$ is irreducible over the field with two elements.
- Write down all 8 elements of the field $\mathbb{F}_2[x]/(x^3 + x + 1)$.
- Show that x is a primitive element, i.e. that the x^i generate all nonzero field elements.
- From this describe the Hamming code Ham(3, 2).