Problem sheet 10

## Ex. 1

Show that the Latin squares $L_{1}$ and $L_{3}$ of order 4 are not orthogonal. Construct two orthogonal Latin Squares of order 4 as follows:
Construct the field of 4 elements by taking all polynomials with coefficients 0 and 1 , and reduce modulo $x^{2}+x+1$.
Show that the 4 elements are: $0,1, x, x+1$.
Write down the addition and multiplication table of this field. From this find two MOLS of order 4. Bring them into standard form.

## Ex. 2

- Show that $f(x)=x^{3}+x+1$ is irreducible over the field with two elements.
- Write down all 8 elements of the field $\mathbb{F}_{2}[x] /\left(x^{3}+x+1\right)$.
- Show that $x$ is a primitive element, i.e. that the $x^{i}$ generate all nonzero field elements.
- From this describe the Hamming code $\operatorname{Ham}(3,2)$.

