Representation growth and the congruence subgroup property

Let G be a finitely generated group. For each n we define a_n to be the number of index n subgroups of G (this is always finite). The theory of subgroup growth deals with the behaviour of the a_n. If the growth rate of the a_n is polynomially bounded, the expression $\sum_{n=1}^{\infty} a_n n^{-s}$ converges for suitable values of the complex parameter s, giving a meromorphic function of s. For example, if G is the integers, one obtains the Riemann zeta function. Subgroup growth has been used as a tool to investigate the properties of infinite groups.

A new idea is to consider instead the representation growth of G: here we look at the growth rate of b_n, the number of irreducible complex n-dimensional characters of G. Recently the speaker and Alex Lubotzky have studied the representation growth of arithmetic groups; roughly speaking, an arithmetic group is an abstract group G together with extra arithmetic data that comes from an embedding of G into a group of matrices over a number field. We conjecture that if G is a semisimple arithmetic group then G satisfies the so-called congruence subgroup property if and only if the growth of the b_n is polynomially bounded.

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