STAND-ALONE PROJECT - FINAL REPORT

Project title Asymptotic Properties of Random Walks on Graphs

Project leader Univ.Prof. Dipl.Ing. Dr. Wolfgang WOESS

P-15577-N05 **Project number**

1. Summary for public relations work

A random walk is a random process on a graph (network), where a particle (walker) moves randomly from point to point. Randomness is provided by a transition matrix which encodes the probabilities to move to a next point, given the actual position. These probabilities are assumed to be adapted to the underlying graph structure by some conditions that have to be specified from case to case. The general theme of the project has been the study of the interplay between structural (geometric, combinatorial, or algebraic) properties of the graph, which is assumed infinite here, and probabilistic as well as analytic features of the random walk: what can we deduce from the type of structure about the behaviour of the random process, and vice versa, what do properties of the latter tell us about the underlying structure ? The interested reader may look at an introductory article by Woess (in German) and a more advanced overview by Saloff-Coste (published in the Notices of the A.M.S), both available online at *www.math.tugraz.at/~woess/#research* , or also the advanced monograph by Woess, *"Random Walks on Infinite Graphs and Groups",* Cambridge Univ. Press, 2000.

Among the achievements of the project, let me outline the subject of lamplighter random walks. In this case, in addition to the moves on the graph, there is a "lamp" at each point. Initially, all lamps are "off", and when the walker moves along, he chooses randomly at each step whether to switch the lamp where he stands or to leave it as it is. In order to understand the process, one has to consider a higher level "lamplighter graph" where one keeps track both of the actual walker's position and the configuration of lamps that are switched on. In the typical situation when the base graph is a two-way-infinite line, we made a little breakthrough thanks to a precise understanding of the geometry of the associated lamplighter graph: it is a Diestel-Leader graph, horocyclic product of two homogenous trees.

This allowed us to describe precisely the harmonic functions of the random walk (i.e., its potential theory) via the Martin boundary, to introduce an explicit method for computing the spectrum of the transition operator, and the asymptotic behaviour of n-step transition probabilities. This was done for Diestel-Leader graphs in general, and subsequently, we introduced horocyclic products of an arbitrary number of trees and exhibited many of their interesting features. One of them is that the transition operator always has pure point spectrum.

Further highlights of the project concern cogrowth and non-backtracking random walks, and asymptotics of random walks on trees with a "fractal" structure.

Eine Irrfahrt (random walk) ist ein Zufallsprozess auf einem Graphen (Netzwerk), wo sich ein Partikel (walker) zufällig von Punkt zu Punkt bewegt. Der Zufall wird hierbei durch eine Übergangsmatrix beschrieben. Bei gegebener aktueller Position gibt diese die Wahrscheinlichkeiten an, mit denen sich das Partikel im nächsten Schritt zu einem der anderen Punkte bewegt. Es wird vorausgesetzt, dass diese Übergangswahrscheinlichkeiten an die zugrunde liegende Graphenstruktur durch fallweise zu spezifizierende Bedingungen angepasst sind. Das generelle Thema des Projektes ist das Studium des Zusammenhanges zwischen (geometrischen, kombinatorischen oder algebraischen) Eigenschaften des Graphen (der hier stets unendlich viele Punkte hat) und probabilistischen sowie analytischen Eigenschaften der Irrfahrt: was können wir aus der Art der Struktur über das Verhalten des Zufallsprozesses schließen, und umgekehrt, was folgern wir aus Eigenschaften des letzteren über die zugrunde liegende Struktur ? Der interessierte Leser kann einen Einführungsartikel von Woess sowie einen anspruchsvolleren Überblick von Saloff-Coste (publiziert in den Notices of the A.M.S.) lesen, die beide online zugänglich sind unter *www.math.tugraz.at/~woess/#research* , oder die Monographie von Woess, *"Random Walks on Infinite Graphs and Groups",* Cambridge Univ. Press, 2000.

Von den Resultaten des Projektes möchte ich hier das Thema der "lamplighter" Irrfahrten näher beschreiben. Zusätzlich zu der Bewegung auf dem Graphen befindet sich in jedem Punkt eine "Lampe". Anfangs sind alle Lampen "ausgeschaltet", und während sich das Partikel (der "Lampenanzünder") bewegt, wird die Lampe am aktuellen Punkt nach dem Zufallsprinzip ein- oder ausgeschaltet.Um diesen Zufallsprozess zu beschreiben, muss man einen übergeordneten "lamplighter"-Graphen betrachten, der sowohl die aktuelle Position des Partikels beschreibt wie auch die Konfiguration der gerade eingeschalteten Lampen. In der typischen Situation wo der Basisgraph der zweseitig unendliche Pfad ist, gelang im Projekt ein kleiner Durchbruch dank einem genauen Verständins der Geometrie des zugeordneten "lamplighter"-Graphen: Es ist ein Diestel-Leader-Graph, das horozyklische Produkt zweier homogener Bäume. Das hat es uns möglich gemacht, die harmonischen Funktionen (d.i., die Potentialtheorie) der Irrfahrt über den Martinrand genau zu beschreiben. Weiters haben wir eine explizite Methode erarbeitet, um das Spektrum des Übergangsoperators und die Asymptotik der n-Schritt-Übergangswahrscheinlichkeiten zu berechnen. Dies wurde für Diestel-Leader-Graphen gemacht, und in der Folge haben wir horozyklische Produkte einer beliebigen Zahl homogener Bäume eingeführt und eine Vielzahl interessanter Eigenschaften nachgewiesen. Eine ist, dass der Übergangsoperator immer ein pures Punktspektrum hat.

Weitere "highlights" des Projektes betreffen Co-Wachstum von Graphen und Irrfahrten ohne Umkehrschritte, sowie die Asymptotik von "random walks" auf einer Klasse von Bäumen mit "fraktaler" Struktur.

2. Brief project report

2.1 Report on the scientific work

2.1.1 Overall scientific concept and goals

I recall briefly the main line of the project proposal, which is still available under *http://www.math.tugraz.at/~woess/fwf.ps* . The basic theme has been the study of the interplay of probabilistic and analytic properties of random walks with the structure theory of the underlying state space (typically an infinite graph or group). This follows the main spirit of the - quite successful - monograph by Woess, Random Walks on Infinite Graphs and Groups (Cambridge Univ. Press, 2000), with emphasis on a set of specific sub-topics. Those outlined in the proposal were random walks on: (A) trees with finitely many cone types, (B) context free graphs, (C) free products and transitive graphs with infinitely many ends, (D) products of certain trees, (E) the Diestel-Leader graphs. A detailed description of the respective achievements will follow below, but it can be anticipated that the work on topic (E) and more or less directly related questions was highly successful, leaving the other themes somewhat behind.

In addressing the question "was there a change of direction in the field between the start and the end of the project?", the answer is "no" as far as the general subject is concerned, but "yes" in the detailed sub-topics. The reason is two-fold. First, new ("hot") topics were of course not neglected, but rather - in correspondence with the competences of the involved persons - included in the research. Second, in the employment of project personnel it has been my explicit strategy to give preference to the best candidates and their competences, possibly at the cost to make slight deviations from the initially proposed tracks and to include additional aspects. Specifically, this concerns primarily the collaboration on cogrowth and random walk with R. Ortner on one hand, and an increased emphasis on aspects of Harmonic Analysis in the work of M. Neuhauser.

2.1.2 Duration, personnel, results, and highlights of the project

(The FWF guidelines suggest to separate those points, but the evolution of the project makes it preferable to maintain a strong link between personnel and description of the results.)

The project started on October 1, 2002 and ended on July 15, 2006.
Due to the nature of the research, it was not necessary to buy any larger equipment. The necessary computer equipment could be provided by the host institute.

Regarding the project staff, the original plan had been to employ Wilfried HUSS as a PhD within this project. However, there had been a delay, first because of lasting health problems, and then because he had to do his civil service. In the meantime, Huss has finished an excellent Diploma (=Master) Thesis on Internal Diffusion Limited Aggregation and is now working as a PhD in the follow-up project (P19115-N18). Since no other outstanding PhD student was in sight, I chose to (cautiously) look for other PostDocs instead.

The project funding was for one PhD and one PostDoc for 3 years each. The designated PostDoc was Sara BROFFERIO who had written her Master thesis under my supervision at the University of Milano before going to Univ. Paris 6, where she got her PhD. Sara is an outstanding young mathematician. After 5 months work within the project P15577-N05, Sara succeeded in being assigned an EU Marie Curie Fellowship for the period 1 March 2003 – 31 August 2004, again hosted by myself at the Institut für Mathematische Strukturtheorie (Mathematik C). Thus, her cooperation in the research group was maintained, while at the

same time allowing to use the FWF funds for additional project staff. The interaction was so intense that from the point of view of the research it is hard to separate the achievements of the two projects precisely. The initial work of Brofferio built upon the results in her thesis and completed the detailed understanding of properties of random walks on the affine group of the hyperbolic plane [19], [1][1] and its discrete counterpart, the homogeneous tree [2]. The latter work, together with my own paper [4], was crucial for the understanding of random walks on lamplighter groups and DL-graphs [7], [22]. Our joint paper **[7]** is, in my view, one of the higlights of the project outcomes. (The research was done in the transitory phase between Sara's collaboration in FWF P15577 and her Curie fellowship.) In this paper, we determine the full *Martin compactification* of a class of random walks on DL-graphs, in particular lamplighter groups. This is the first such result for random walks on discrete groups that are amenable and have exponential growth. It is achieved via precise asymptotic estimates in space of the Green kernel of the random walk, based on a subtle idea of Brofferio how to compare the latter with the Green kernels of the projected random walks on each of the two trees that build up the DL-graph. Let me remark that this work falls precisely into subtopic (E) of the original project proposal.

The note [8] is also related with subtopic (E): it shows how one can compute the spectral radius and the norm of “lamplighter” transition operators on a variety of groups, resp. transitive graphs. This is based on my substantial collaboration **[25]** (plus a previous paper of 1997) with L. SALOFF-COSTE. This started before the project FWF P15577 and cannot be directly attributed to the latter, although it is linked with the basic approach to random walks on the affine group of a tree, and had a strong influence on **[15].**

When it became clear that no excellent candidate for the PhD position was in immediate sight, I looked for another PostDoc, and found Dr. Ronald ORTNER who had written a Thesis in Discrete Geometry. He had a 50% project position, and a 50% assistant position at the institue. He first worked on subtopic (C) of the proposal (local limit theorem on free products with amalgamation), which however turned out to lead to unexpected and serious difficulties. Answering a ``hot'' question posed by group theorists, he then studied jointly with me **[12]** a way how to extend the notion of *cogrowth* in a satisfactory way to general (not necessarily regular) graphs via non-backtracking random walks. Non-backtracking RW is interpreted as a Markov chain on the associated line graph of oriented edges, and the structure of the latter is compared with that of the original graph. In this setting, we obtain all results that were previously known only for regular graphs, comprising Grigorchuk’s theorem regarding groups (=Cayley graphs), by a completely new approach that combines Markov chain theory with quasi-isometry and a certain amount of Functional Analysis. This is also linked with the covering tree of the underlying graph, and can be seen in relation with subtopic (A) of the proposal, although somewhat deviating from the original plans. In any case, I believe that **[12]** is another highlight of the achieved results.

When it became clear that Ortner would move to a regular assistant position at Montanuniv. Leoben, I once more advertised a PostDoc position. The (by far) best candidate was Dr. Markus NEUHAUSER, with a PhD from TU München and a one year PostDoc at Univ. Neuchatel, working mostly in Harmonic Analysis (Kazdhan constants and other aspects of representation theory of finitely generated groups). The relation with random walk themes is obvious (mainly via Spectral Theory), so that I considered it preferable to employ the best candidate at the cost of shifting part of the research towards those topics. Among other, Neuhauser’s paper [6] concerns wreath products, i.e., general “lamplighter groups” like those which have the DL-graphs of subtopic (E) as their Cayley graphs. Of course, I gave Neuhauser the liberty to carry on working in this area, with respectable outcomes [3], [6], [9], [10]. At the same time, we started an interesting an fruitful collaboration with Prof. Laurent BARTHOLDI (Lausanne). This had been preceded by my work [5] with Bartholdi, where we

[1] References refer to list of publications in §4 below.

found a very satisfactory “elementary” approach to the completely explicit computation of the spectrum of transition operators on DL-graphs (in particular, lamplighter groups), that is, the diagonalization of the transition operator. The main outcoume **[15]** of the collaboration of Bartholdi, Neuhauser and myself is the third of the highlights of this project. With the definition of the DL-graphs as the “horocyclic products” of two trees as the starting point, we introduce the horocyclic product of an arbitrary number of homogeneous trees. We then study a variety of properties of the resulting graph: we determine the full isometry group and show that it is unimodular precisely when all trees have the same degree. Thus, it can be a Cayely graph only in the latter case, and for a very large class of cases, following the spirit of the construction of discrete subgroups of Lie groups, we exhibit a finitely generated group whose Cayley graph is the horocyclic product. We also determine the Poisson boundary of random walks on the horocyclic product. Furthermore, we show that the spectrum of the simple random walk is always pure point, extending the results of Grigorchuk and Zuk, Dicks and Schick, and of [5] regarding lamplighter groups. Thus, it appears that there are many groups with Cayley graphs that have pure point spectrum, a property so far considered “unexpected”. As a matter fact, in the post-project paper [30], on the basis of **[15]**, Neuhauser is able to add new substance to the counterexamples to a conjecture of Atiyah. On one hand the work done in **[15]** fits well into subtopic (E) of the proposal, but on the other hand, it goes beyond the specific ideas stated there: a fruitful development of this specific topic.

Along with his collaboration in the project, Neuhauser also brought his former fellow PostDoc from Neuchatel, Dr. Anders KARLSSON (KTH Stockholm) for a very fruitful one-week visit to Graz (the trip was funded by the RDSES program of ESF). This resulted in the two project publications [10] and [13], the latter belonging again to subtopic (E) and its extension to a wider class of structures.

In June 2004, I saw that funds were sufficient to advertise another PostDoc position within the project. Among several candidates, Dr. Florian SOBIECKY was the most promising; he gave a very good talk and showed vivid interst in work that involves more than one single field. Sobieczky brought new topics into the project, thus leading to a slight deviation from the original proposal: his main interest is in random walks on percolation clusters and similar random graphs. On the basis of his PhD Thesis (Univ. Göttingen), and with some additional suggestions by myself, he has concluded the paper [14]. He has developed an interesting comparison technique for Markov chains by *interlacing* of their eigenvalues. This technique has further potential application to random walks on finite graphs; in [14], it is applied to RW on finite percolation clusters. He has also started to work on DL-graphs - again subtopic (E) - considering the isoperimetry of percolated horocyclic products of trees. This work is still unfinished. Sobieczky is an open-minded and interested character who brought a lively interaction with staff and guests into the project, even though one may observe that it takes him a certain effort to bring his various research ideas to an end.

A year before the end of the project, Dr. Elmar TEUFL was employed for 6 months on a 50% basis in the project. Teufl had written his PhD Thesis under the supervision of my colleague Peter Grabner (Institute of Analysis and Computational Number Theory, Math A) on topics that are part of my own research field, based on my collaboration with Grabner, dating back more than 10 years, on random walk on *fractal graphs;* see e.g. my survey article [18]. Grabner is, among other, a specialist in the use of complex analytic methods in combinatorial enumeration, in the spirit of Flajolet and Odlyzko. Teufl has taken up this topic very successfully and is extremely skillful in dealing with complex dynamics and functional equations arising from random walk and other, more combinatorial questions. Previously, he had held a project position in Grabner’s START project, and also a 50% assistant position at Institut für Mathematische Strukturtheorie (Math C), maintaining a permanent fruitful interaction with me. I had supported his application for a EU Curie fellowship under the supervision of A. Grigor’yan at Bielefeld University In the brief waiting time until the start of that fellowship, I was glad to host him in FWF P15577. During that period, he has

contributed the papers [16] and **[17]** to the project publications. **[17]** falls basically into the subtopic (A), although the trees under consideration, rather than having finitely many cone types, have a "fractal" structure. The "nice trees" appeared in the 1984 book of Doyle and Snell, and asymptotics of return probabilities were later studied in a non-rigorous way by theoretical physicists (Burioni, Cassi, Pirati and Regina). Here, Teufl and B. Steinsky (another former PhD student of P. Grabner) give a rigorous treatment of those asymptotics, which is a rather difficult task because of the multi-dimensional functional equation that arises. Once more, the asymptotics exhibit the interesting phenomenon of logarithmic oscillations. This is another highlight among the project publications, although it should be added that a final piece of the "puzzle" regarding the treatment of the functional equation was inserted by Teufl only recently.

In the final 6 months of the project, the line of research of F. Sobieczky received additional support from Dr. Adam TIMAR, a young PhD of Hungarian nationality from the excellent school of Russell Lyons (Bloomington, Indiana). His work concerned percolation on vertex-transitive graphs, and in particular, non-unimodular graphs [11]. This meant of course a deviation from the original plans. The link with the project themes has it roots in a paper of Soardi and Woess [Math. Zeitschrift 205 (1990) 471-486], where we related amenability and the modular function of the isometry group of a transitive graph with the spectral radius of simple random walk (this has been taken up in **[25]**). The interplay and common ways of thinking between random walk and percolation theory on general transitive graphs has always been fascinating for me, and I was very glad to be able to incorporate part of this into the FWF project.

Some final words are due on the external project collaborators. Five prominent names were included in the proposal, and each of them came to Graz at least once. It has been my strategy to see whether those visits could be realized on the basis of other funding whenever possible. Indeed, each of them came as a one-month visiting professor funded by TU Graz, thereby also giving an advanced course: Dr. Tatiana NAGNIBEDA-SMIRNOVA (KTH Stockholm, now Univ. Geneva) already shortly before the very beginning of the project, Prof. Laurent SALOFF-COSTE (Cornell Univ., Ithaca) in January 2003, Prof. Andrzej ZUK (ENS Lyon, now Univ. Paris 6) in March 2004, Prof. Laurent BARTHOLDI (UC Berkeley, now EPF Lausanne) in October 2004, and Prof. Vadim A. KAIMANOVICH (IU Bremen) in January 2006.

In addition, visits of KAIMANOVICH in October 2002 and BARTHOLDI in June 2003 were funded by the project. Indeed, those two had a particularly positive influence on the project work. Kaimanovich is always very generous in giving hints and suggestions on the basis of his profound mathematical knowledge. In particular, he has had a very good interaction with Sobieczky, although the resulting research has not yet been concluded. Bartholdi is a brilliant young mathematician, open minded and with multiple interests, who interacted with all team members that he met during his stays. Papers [5] and **[15]** (with a total of 69 pages) are the visible and substantial outcome of this interaction.

In conclusion, let me remark that an overall presentation of the activities, guests, publications, and so on, of my research group at Graz since 1999 is available under *http://www.math.tugraz.az/~woess/mathCreport.pdf*

2.2. Personnel development – importance of the project for the scientific careers of those involved (including the project leader)

(2000 characters excl. spaces)

• Dr. Sara BROFFERIO worked within the project FWF P15577 in the period 1 October 2002 – 28 February 2003. From 1 March 2003 until 31 August 2004 she held a Curie fellowship at

the same host institute. Starting with 1 September 2004, she now has a permanent maitre de conference position at Univ. Paris-Orsay, France.

• Dr. Roland ORTNER worked within FWF P15577 in the period 1 October 2002 – 31 July 2003. Since then, he holds a regular assistant position at the Department of Mathematics and Information Technology of Montanuniversität Leoben, Austria.

• Dr. Markus NEUHAUSER worked within FWF P15577 in the period 1 October 2003 – 30 September 2005. He then moved to an assistant position at the Institute of Mathematics, Univ. Göttingen, Germany.

• Dr. Florian SOBIECKY worked within FWF P15577 in the periods 1 October 2004 – 31 August 2005 and 1 January – 30 April 2006, interrupted for the purpose of a stay at Univ. Bochum (Germany). Since 1 October 2006 he is continuing to work at Institut für Math. Strukturtheorie within a new, individual FWF project of his own.

• Dr. Elmar TEUFL worked within FWF P15577 in the period 1 April – 30 September 2005. He then moved to Bielefeld University, Germany, for a Curie Fellowship after the end of which he will get an assistant position at the Faculty of Mathematics at Bielefeld.

• Dr. Adam TIMAR worked within FWF P15577 in the period 15 January – 15 July 2006. He now holds an assistant professor position at University of British Columbia, Vancouver, Canada.

One can see that project collaborators have spread out over the world and are continuing their research career.

Regarding the effects on the host institute, and on myself as the project leader, I first want to remark that with the neo-liberal ideology sweeping through European Universities, my University's primary way to measure research success of its academic staff is via the amount of raised funds, as opposed to the effective contents of the research. Thus, in order to develop and maintain a reasonable research environment, funded projects such as the present one have become essential. Apart from this somewhat sarcastic remark, there are visible, truly positive effects of that project: it laid the base for building up an active research group in my field after my return to Austria in 1999. My international visibility as well as that of the group has clearly increased. Maintaining good contacts with all project collaborators, I could amplify my already existing good international contacts and prepare the ground for new collaboration.

Graz, 21 November 2006, Wolfgang Woess

3. Information on project participants

not funded by the FWF			**funded by the FWF (project)**		
co-workers	**number**	**Person-months**	**co-workers**	**number**	**Person - months**
non-scientific co-workers	1 (Secretary)	2	non-scientific co-workers		
diploma students			diploma students		
PhD students			PhD students		
post-doctoral co-workers	2 (Ortner, Karlsson)	3.5	post-doctoral co-workers	6 (Brofferio, Ortner, Neu-hauser, Sobieczky, Teufl, Timar)	56
co-workers with “Habilitation” (professorial qualifications)	1 (Zuk)	1	co-workers with “Habilitation” (professorial qualifications)		
Professors	3 (Woess; Kaimano-vich, Saloff-Coste)	24	professors	2 (Woess, Kaimano-vich)	1

Remark: it is not completely clear whether (e.g.) I myself (Woess) should figure as “funded”, since I did not use any project money, except for one conference participation.

4. Attachments

List 1 - Scientific publications

All project publications that have appeared, resp. have been accepted for publication are peer-reviewed. Papers that are currently submitted as well as those which are still being brought to paper are / will also be subject to a peer-reviewing procedure.
Some of the publications (in a transitory phase of project staff, or in the case of external co-authors) are also attributed to other projects, resp. funding.

1.1. Project publications that have already appeared in print

[1] S. Brofferio: *Speed of stochastic locally contractive systems,* Ann. Probab. 31 (2003), 2040-2067.

[2] S. Brofferio: *Renewal theory on the affine group of an oriented tree,* J. Theoret. Probab. 17 (2004) 819-859.

[3] A. Karlsson and M. Neuhauser: *The Bass conjecture and growth in groups,* Colloquium mathematicum 100 (2004) 23-27.

[4] W. Woess: *Lamplighters, Diestel-Leader graphs, random walks, and harmonic functions,* Combinatorics, Probability & Computing 14 (2005) 415-433.

[5] L. Bartholdi and W. Woess: *Spectral computations on lamplighter groups and Diestel-Leader graphs,* J. Fourier Analysis Appl. 11 (2005) 175-202.

[6] M. Neuhauser: *Relative property (T) and related properties of wreath products,* Mathematische Zeitschrift 251 (2005) 167-177.

[7] S. Brofferio and W. Woess: *Green kernel estimates and the full Martin boundary for random walks on lamplighter groups and Diestel-Leader graphs,* Annales Inst. H. Poincaré (Probab. & Stat.) 41 (2005) 1101-1123.

[8] W. Woess: *A note on the norms of transition operators on lamplighter graphs and groups,* Int. J. Algebra and Computation 15 (2005) 1261-1272.

[9] M. Neuhauser: *Kazhdan constants for compact groups,* J. Australian Math. Soc. 81 (2006), 11-14.

[10] A. Karlsson and M. Neuhauser: *Heat kernels, theta identities, and zeta functions on cyclic groups,* Contemporary Mathematics 394 (2006), 177-189.

[11] A. Timar: *Percolation on nonunimodular graphs,* Annals Probab. 34 (2006), ?-?.

1.2. Project publications that have been accepted for print

[12] R. Ortner and W. Woess: *Non-backtracking random walks and cogrowth of graphs,* Canadian J. Math., to appear, 16 pages.

[13] A. Karlsson and W. Woess: *The Poisson boundary of lamplighter random walks on trees,* Geometriae Dedicata, to appear, 13 pages.

1.3. Project publications that have been submitted

[14] F. Sobieczky: *Random walks on the finite components of random partial graphs of transitive graphs,* preprint, TU Graz (2005) [currently under revision following referees' suggestions], 22 pages.

[15] L. Bartholdi, M. Neuhauser and W. Woess: *Horocyclic products of trees,* preprint, TU Graz (2006), 41 pages.

[16] E. Teufl and S. Wagner: *Enumeration problems for classes of self-similar of graphs,* preprint, TU Graz (2006), 19 pages.

1.4. Project publication in phase of final preparation

[17] B. Steinsky and E. Teufl: *Random walk on nice trees,* TU Graz & Univ. Bielefeld (2006), 26 pages.

1.5. Publications having close relation with the project

In the period of the FWF project P15577-N05, the research team at Institut für Mathematische Strukturtheorie (Math. C) has produced a number of further publication that did not directly rely on the FWF funding, but are – as regards to contents and involved persons – closely related with the project, so that the outcomes cannot be seen as completely separated from the project work. These publications are listed here.

[18] W. Woess: *Generating function techniques for random walks on graphs,* in *"Heat Kernels and Analysis on Manifolds, Graphs, and Metric Spaces"*, P. Auscher, Th. Coulhon and A. Grigor'yan, eds. Contemporary Math. 338 (2003) 391-423

[19] S. Brofferio: *How a centred random walk on the affine group goes to infinity,* Ann. Inst. H. Poincaré (Probab. &Stat.) 39 (2003) 371-384.

[20] B. Krön and E. Teufl: *Asymptotics of the transition probabilities of the simple random walk on self-similar graphs,* Trans. Amer. Math. Soc. 356 (2004) 393-414.

[21] D. I. Cartwright and W. Woess: *Isotropic random walks in a building of type Ã_d* , Math. Zeitschrift 247 (2004) 101-135.

[22] S. Brofferio and W. Woess: *Positive harmonic functions for semi-isotropic random walks on trees, lamplighter groups, and DL-graphs,* Potential Analysis 24 (2006) 245-265.

[23] S. Brofferio: *"The Poisson boundary of finitely generated groups of rational affinities",* J. Math. Sci., to appear.

[24] S. Brofferio: *"The Poisson boundary of random rational affinities",* Ann. Inst. Fourier (Grenoble) 56 (2006) 499-515.

[25] L. Saloff-Coste and W. Woess: *Transition operators on co-compact G-spaces,* Revista Matematica Iberoamericana, to appear, 56 pages.

[26] S. Blachère and S. Brofferio: *Internal diffusion limited aggregation on groups having exponential growth,* Probability Theory Rel. Fields, to appear, 20 pages.

[27] D. I. Cartwright and W. Woess: *The spectrum of the averaging operator on a network (metric graph),* Illinois J. Math., to appear, 25 pages.

[28] L. Gilch: *Rate of escape of random walks on free products,* J. Australian Math. Soc., to appear, 22 pages.

[29] L. Gilch: *Rate of escape of lamplighter random walks on trees,* preprint, TU Graz (2005), 16 pages.

[30] M. Neuhauser: *New counterexamples to the strong Atiyah conjecture*, preprint, Univ. Göttingen (2006).

List 2 - Project-related participation in international scientific conferences

The conference participations marked with **(*)** were funded by the project. For the other conference participations, different funding was found.

2.1. Conference participations - invited lectures

1) W. Woess: *"Generating function techniques for random walks on graphs",* series of talks (17. April – 2. May 2002) in the special trimester "Heat Kernels, Random Walks & Analysis on Metric Spaces", Centre Emile Borel, Paris (France), April-July 2002.

2) W. Woess: *"Transition operators on co-compact G-spaces, and a theorem of Brooks",* invited talk, conference "Analysis on Graphs and Metric Spaces", Institut Henri Poincaré, Paris (France), 26-28 June 2002.

3) W. Woess: *"Lamplighters, Diestel-Leader graphs, random walks, and a quasi-isometry problem",* International Conference on Group Theory, Gaeta (Italy), 1-6 June 2003.

4) W. Woess: *"Transition operators on co-compact G-spaces"* ,Workshop on Groups, Geometry and Dynamics (in memory of Bob Brooks), Technion, Haifa (Israel), 5-9 January 2004.

5) M. Neuhauser: *"The metaplectic representation on finite Heisenberg groups",* Third Viennese Gabor Workshop, Univ. Vienna (Austria), 10-12 May 2004. **(*)**

6) W. Woess: *"Lamplighter random walks",* Third Colloquium on Mathematics and Computer Science, TU Vienna (Austria), 13.09.2004 bis 17.09.2004. (Talk within the Special Day for Helmut Prodinger.)

7) W. Woess: *"Non-backtracking random walks and cogrowth of graphs",*XXV Conferenza di Analisi armonica, Bologna (Italy), 6-9 April 2005.

8) W. Woess: *"Horocyclic products of trees",* Minisymposium "Symmetries of finite and infinte graphs, Montanuniv. Leoben (Austria), 19 May 2005.

9) Elmar Teufl: *"Simple random walk on a class of fractal-like trees",* special session "Stochastic Analysis on Metric Spaces", 2nd joint meeting AMS – DMV – ÖMG, Mainz (Germany), 16-19 June 2005. **(*)**

10) W. Woess: *"Martin compactification of random walks on the lamplighter group",* Journées "Marches Aléatoires sur les graphes et les groupes", Univ. Tours (France), 3-5 July 2006.

11) W. Woess: *"Lamplighter random walks",* Fourth Colloquium on Mathematics and Computer Science, Univ. Nancy (France), 18-22 September 2006.

2.2. Conference participations - lectures

1) S. Brofferio: *"Renewal theory on an oriented tree",*conference on "Probability Theory on Algebraic-Topological Structures", Bommerholz (Germany), 3-7 March 2003. **(*)**

2) W. Woess: *“Lamplighter-Irrfahrten und harmonische Funktionen”,* Kolloquium ueber Wahrschinlichkeitstheorie und Statistik, TU Graz, 9-11 April 2003.

3) W. Woess: *“Harmonic functions on lamplighter graphs”,* 8. Österreichischisches Mathematikertreffen, Nachbarschaftstagung in Kooperation mit SIMAI und UMI, Bozen (Italy), 22 -26 September 2003.

4) S. Brofferio: *"Frontiera di Poisson delle transformazioni affini a coefficienti razionali",* Conference on "Geometria algebrica ed aritmetica”, Padova (Italy), 7-9 January 2004.

5) S. Brofferio: *"The Poisson boundary of random rational affinities",* Conference on "Théorie Ergodique des Actions de Groupes”, Orléans (France), 23-25 May 2004.

6) S. Brofferio: *"Martin boundary for Diestel-Leader graphs and lamplighter groups",* Conference on "Géométrie et Analyse sur les Structures Aléatoires”, Lille (France), 25-28 May 2004.

7) S. Brofferio: *"The Poisson boundary of random rational affinities",* Conference on "Geometric Group Theory, Random Walks, and Harmonic Analysis", Cortona (Italy), 13-19 June 2004.

8) M. Neuhauser: *"Relative property (T) and related properties of wreath products",* Conference on "Geometric Group Theory, Random Walks, and Harmonic Analysis", Cortona (Italy), 13-19 June 2004. **(*)**

9) W. Woess: *“Non-backtracking random walks and cogrowth of graphs”,* Conference on "Geometric Group Theory, Random Walks, and Harmonic Analysis", Cortona (Italy), 13-19 June 2004.

10) F. Sobieczky: *"Annealed return probability of a random walk on a stationary random partial graph of the Cayley graph of an amenable group”,*Workshop on "Self-similar Random Structures, Hausdorff Dimension and Branching, EURANDOM, Eindhoven (The Netherlands) 14-18 March, 2005. **(*)**

11) M. Neuhauser: *"Heat kernels on cyclic groups”,* Workshop "Group theoretical methods, operator theory, and non-orthogonal expansions", Erwin Schrödinger Institute, Vienna (Austria), 10-22 April 2005. **(*)**

12) M. Neuhauser: *“Explicit construction of the metaplectic representation for finite cyclic groups”,* Conference on ”Modern methods of time-frequency analysis”, Strobl (Austria), 22-27 May 2005. **(*)**

13) F. Sobieczky: *"Random Walks on the finite components of random partial graphs of transitive graphs”,* First Cornell Summer School in Probability (NSF), Cornell, Ithaca, NY (USA), 10-23 July 2005. **(*)**

14) W. Woess: *“Horozyklische Produkte von Bäumen”,* 16. Internationaler Kongress der Österreichischen Mathematischen Gesellschaft, Klagenfurt, 18-23 September 2005. **(*)**

15) A. Timar: *“Neighboring clusters in Bernoulli percolation”,* RDSES/ESI Educational Workshop on Discrete Probability, Erwin Schrödinger Institute, Vienna (Austria), 13-25 March 2006**. (*)**

16) E. Teufl: *“Spectral dimension and the number of spanning trees of fractal graphs”,* RDSES/ESI Educational Workshop on Discrete Probability, Erwin Schrödinger Institute, Vienna (Austria), 13-25 March 2006.

17) F. Sobieczky: *"Anchored expansion of products of random trees",* RDSES/ESI Educational Workshop on Discrete Probability, Erwin Schrödinger Institute, Vienna (Austria), 13-25 March 2006.

18) A. Timar: “*Neighboring clusters in Bernoulli percolation”,* AMS Meeting, San Francisco, 29-30 April 2006. **(*)**

19) F. Sobieczky: *"Anchored isoperimetry of products of trees”,* 9th International Vilnius Conference on Probability Theory and Statistics Vilnius (Lithuania), 25-30 June 2006. **(*)**

2.3 Conference participations without lectures or posters

1) R. Ortner, conference on ”Probability Theory on Algebraic-Topological Structures”, Bommerholz (Germany), 3-7 March 2003.

List 3 - Development of collaborations

N			Nature	**N**	(national);
				E	(European);
				I	(other international cooperation)
	E		Extent	**E1**	**low** (e.g. no joint publications but mention in acknowledgements or similar);
				E2	**medium** (collaboration e.g. with occasional joint publications, exchange of materials or similar but no longer-term exchange of personnel);
				E3	**high** (extensive collaboration with mutual hosting of group members for research stays, regular joint publications etc.)
		D	Discipline	**D**	within the discipline
				T	transdisciplinary

N	E	D	Collaboration partner / content of the collaboration
E	E3	D	1) Name: Vadim A. Kaimanovich Title: Prof. Dr. Institution: International University Bremen Content: Permanent collaboration with W. Woess since many years on boundary theory and related subjects. Collaboration with F. Sobiecky on amenability questions.
E	E3	D	2) Name: Laurent Bartholdi Title: Prof. Dr. Institution: EPF Lausanne Content: Intense collaboration and joint publications on spectral theory of transition operators on DL-graphs and on horocyclic products of trees
E	E2	D	3) Name: Anders Karlsson Title: Dr. Institution: KTH Stockholm Content: Collaboration and joint publications with M. Neuhauser (growth of groups, heat kernels and zeta functions) and W. Woess (lamplighter random walks on trees)
I	E2	D	4) Name: Laurent Saloff-Coste Title: Prof. Dr. Institution: Cornell University, Ithaca, NY Content: Collaboration and joint publications on norms of transition operators
E	E1	D	5) Name: Nina Gantert Title: Prof. Dr. Institution: Universität Münster Content: First contacts in June 2006 for establishing future collaboration, linked with the follow-up project P19115-N18

List 4 - Organization of international conferences and workshops

In the period of the project, I (co-)organized the following international conferences, where project collaborators were able to participate.

"Geometric Group Theory, Random Walks, and Harmonic Analysis", Conference in Cortona, Italy, 13-19 June 2004.

Special session "Stochastic Analysis on Metric Spaces" at the 2nd Joint Meeting of AMS, DMV, ÖMG, Mainz, Germany, 16-19 June 2005.

"RDSES/ESI Educational Workshop on Discrete Probability", Erwin Schrödinger Insitute, Vienna, 13 – 24 March 2006.

For more details, see *http://www.math.tugraz.at/~woess*

List 5 - Follow-up projects

1*) "Random walks, random configurations, and horocyclic products",* FWF P19115-N18, stand-alone project lead by Prof. Wolfgang Woess at Institut für Mathematische Strukturtheorie of TU Graz, approved and started on 1 October 2006, planned duration: 3 years. Funded positions: 1 PhD, 1 PostDoc, plus international collaboration.

2) *"Random walks on random partial graphs of transitive graphs",* FWF P18703-N18, individual stand-alone project of Dr. Florian Sobieczky at Institut für Mathematische Strukturtheorie of TU Graz, approved and started on 1 October 2006, planned duration: 2 years.