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★**Denumerable Markov chains.**
Generating functions, boundary theory, random walks on trees.
EMS Textbooks in Mathematics.
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This book is about time-homogeneous Markov chains that evolve in discrete times on a countable state space. It gives a clear account on the main topics of the subject and expressly focuses on three goals:

 (1) to give a comprehensive overview on the generating functions of the transition kernel of a Markov chain, explaining their use and interest;
 (2) to give an introduction to the discrete potential theory of transient Markov chains and the associated topological boundary theory;
 (3) to present a broad collection of results on nearest neighbour random walks on trees.

The difficulty level varies from basic to more advanced, addressing audiences ranging from master's degree students to researchers in mathematics. Rigorous proofs are given, exploiting a wide range of tools coming from measure theory, harmonic analysis and combinatorics. The book is enriched by many examples and exercises with solutions.

   The book is organized as follows:

   The first chapter presents Markov chains: first by means of elementary examples and then by a complete measure-theoretic construction. It also introduces transition probabilities and the associated generating functions. Chapter 2 defines irreducible classes and the period of a Markov chain. It also gives the first results on the asymptotic behavior of the transition probabilities in terms of the spectral radius.

   The classification of Markov chains in terms of transience vs. recurrence is treated in Chapter 3. This chapter also extensively covers theorems on convergence to the stationary measure for positive recurrent chains (with finite or infinite state space), with an excursion on matrix analysis. The last two sections are devoted to ergodic theorems and ρ-recurrence.

   Chapter 4 is devoted to reversible Markov chains: their relation to networks, speed of convergence to the stationary measure (with emphasis on the special case of random walks on groups) and recurrence criteria.

   Chapter 5 investigates three interesting models that can be interpreted as population evolutions: birth-and-death chains, Galton-Watson trees and branching Markov chains. The latter models a population where each individual moves on a graph according to the law of a Markov chain and reproduces at the same time according to a Galton-Watson tree. Such processes are intimately related to generating functions and reveal another aspect of their probabilistic interest.

   Chapter 6 is an introduction to discrete potential theory for transient Markov chains. In the first section it focuses on finite graphs and solves the Dirichlet problem for harmonic functions in

probabilistic terms. Further sections treat infinite graphs, presenting the main classical results (the existence of a positive superharmonic function, the relation with first entrance stopping times, the Riesz decomposition theorem, the domination principle, etc.).

Chapter 7 gives a detailed, but accessible, exposition of Martin boundary theory. After presenting the construction of the Martin compactification (the smallest compactification that makes the Martin kernel continuous), it proves convergence to the boundary and Poisson-Martin integral representation of positive harmonic functions. The last section is devoted to the representation of bounded harmonic functions, Poisson boundary and its interpretation in terms of terminal random variables.

After a brief Chapter 8, on minimal harmonic functions for random walks on the integer grids, the long Chapter 9 is dedicated to nearest neighbor random walks on trees (finite or infinite, not necessarily locally finite). Among other results the author gives a geometric construction of the Martin boundary, proves an integral representation for *all* (not necessarily positive) harmonic functions, gives criteria for transience/recurrence for trees with finitely many cone types, and studies in detail rate of escape and spectral radius.

Reviewed by *Sara Brofferio*