Graz University of Technology

Institut für Diskrete Mathematik

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Cohomology groups of random simplicial complexes

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Given a dimension $k \geq 2$ and a probability p, define the binomial random k-dimensional simplicial complex $\mathcal{G}_p$ as the downward-closure of the binomial random $(k+1)$-uniform hypergraph, in which each hyperedge is present with probability p independently. For each $j \leq k$, call a k-dimensional simplicial complex $\mathbb{F}_2$-*cohomologically j-connected* if all cohomology groups for dimensions 1 up to j with coefficients in the two-element field $\mathbb{F}_2$ vanish, and if furthermore the zero-th cohomology group is isomorphic to $\mathbb{F}_2$.

For each $j \leq k$, we prove the existence of a sharp threshold for $\mathbb{F}_2$-cohomological j-connectedness of $\mathcal{G}_p$. A similar result has been proved for a different model of random complexes by Linial and Meshulam (2006) and by Meshulam and Wallach (2009). In addition, we prove a hitting time result, relating $\mathbb{F}_2$-cohomological j-connectedness with the disappearance of the last minimal obstruction. As a corollary, we deduce an analogous hitting time result for the Linial-Meshulam model, a result which has previously only been known for $k = 2$. Finally, we determine the limiting probability for $\mathbb{F}_2$-cohomological j-connectedness when p lies in the critical window around the threshold.

In this talk, we focus on the main intuition and proof ideas behind these results.

Joint work with Oliver Cooley, Nicola del Guidice, and Mihyun Kang.

Mihyun Kang