Embedding large minors in weak expanders and in sparse random graphs

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A graph G on n vertices is called an α-expander if the external neighborhood of every vertex subset U of size $|U| \leq n/2$ in G has size at least $\alpha|U|$.

Extending and improving the results of Plotkin, Rao and Smith, and of Kleinberg and Rubinfeld from the 90s, we prove that for every $\alpha > 0$, an α-expander G on n vertices contains every graph H with at most $cn/\log n$ vertices and edges as a minor, for $c = c(\alpha) > 0$. Alternatively, every n-vertex graph G without sublinear separators contains all graphs with $cn/\log n$ vertices and edges as minors. Consequently, a supercritical random graph $G(n, (1+\epsilon)/n)$ is typically minor-universal for the class of graphs with $cn/\log n$ vertices and edges. The order of magnitude $n/\log n$ in the above results is optimal.

A joint work with Rajko Nenadov.

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