11a) $\sum_{n\geq 1} \left|\frac{\cos(2n)-1}{3^n}\right| \leq \sum_{n\geq 1} \frac{2}{3^n} \leq 2\cdot \sum_{n\geq 0} \frac{1}{3^n} = \frac{2}{1-\frac{1}{3}} = \frac{2}{\frac{2}{3}} = \underline{\underline{3}}$

$|\cos(2n)-1| \leq 2$ da $\cos(2n) \in [-1,1]$

Also: $\sum_{n\geq 1} \frac{\cos(2n)-1}{3^n}$ absolut konvergent

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11b) $\sum_{n=2}^{\infty} \frac{\sqrt{n^2+n}}{\sqrt{n^2-1}\,(n+1)} = \sum_{n=2}^{\infty} \frac{\sqrt{n}\sqrt{n+1}}{\sqrt{n^2-1}\,(n+1)} = \sum_{n=2}^{\infty} \frac{\sqrt{n}\,\cancel{\sqrt{n+1}}}{\cancel{\sqrt{n+1}}\sqrt{n-1}\,(n+1)}$

$= \sum_{n=2}^{\infty} \underbrace{\sqrt{\frac{n}{n-1}}}_{\geq 1} \cdot \frac{1}{n+1} \geq \sum_{n=2}^{\infty} \frac{1}{n+1} = \sum_{n=1}^{\infty} \frac{1}{n} = \infty$

$\Rightarrow \sum_{n=2}^{\infty} \frac{\sqrt{n^2+n}}{\sqrt{n^2-1}\,(n+1)}$ divergent

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12c) $\sum_{n=1}^{\infty} \frac{3^{5n}}{(2n+1)!}$

Quotientenkriterium: $\frac{\frac{3^{5(n+1)}}{(2(n+1)+1)!}}{\frac{3^{5n}}{(2n+1)!}} = \frac{\frac{3^{5n+5}}{(2n+3)!}}{\frac{3^{5n}}{(2n+1)!}} = \frac{3^{5n+5}\,(2n+1)!}{3^{5n}\,(2n+3)!}$

$= 3^5 \cdot \frac{1}{(2n+3)(2n+1)} \xrightarrow{n\to\infty} 0 = q < 1$

$\Rightarrow$ Reihe absolut konvergent

13a) $\underbrace{\sum_{n=1}^{\infty} \frac{1}{\sqrt{(n+1)(2n-1)}}}_{(*)} \geq \sum_{n=1}^{\infty} \frac{1}{\sqrt{(n+1)(2n+2)}} = \sum_{n=1}^{\infty} \frac{1}{\sqrt{2}} \frac{1}{n+1} = \infty \Rightarrow (*)$ divergent

14) $\sum_{n=0}^{\infty} \left[\frac{6}{5^n} - \frac{2}{3^{n+1}} + \frac{(-1)^n}{7^n}\right] = 6\cdot\sum_{n=0} \left(\frac{1}{5}\right)^n - \frac{2}{3}\cdot \sum_{n=0} \left(\frac{1}{3}\right)^n + \sum_{n=0} \left(-\frac{1}{7}\right)^n$

$= \frac{6}{1-\frac{1}{5}} - \frac{2}{3}\frac{1}{1-\frac{1}{3}} + \frac{1}{1+\frac{1}{7}} = \frac{15}{2} - 1 + \frac{7}{8} = \underline{\underline{\frac{59}{8}}}$