16(g)

$$\frac{2^n}{n!} = \frac{2\cdot\ldots\cdot 2}{1\cdot 2\cdot\ldots\cdot(n-1)\cdot n} = \underbrace{\frac{2}{1}}_{=2}\cdot\underbrace{\frac{2}{2}}_{=1}\cdot\frac{2}{3}\cdot\underbrace{\frac{2}{4}}_{\le\frac{2}{3}}\cdot\ldots\cdot\underbrace{\frac{2}{n}}_{\le\frac{2}{3}} \le 2\cdot 1\cdot\frac{2^{n-2}}{3^{n-2}} \to 0$$

16(k)

$$\frac{1}{n(n+1)}+\ldots+\frac{1}{(2n-1)(2n)} = \sum_{k=n}^{2n-1}\frac{1}{k(k+1)} = \sum_{k=1}^{2n-1}\frac{1}{k(k+1)} - \sum_{k=1}^{n-1}\frac{1}{k(k+1)}$$

In der Vorlesung wurde gezeigt:

$$\sum_{k=1}^{n}\frac{1}{k(k+1)} = \frac{n}{n+1}$$

Damit:

$$\frac{1}{n(n+1)}+\ldots+\frac{1}{(2n-1)(2n)} = \frac{2n-1}{2n} - \frac{n-1}{n} = \frac{2n^2-n-2n^2+2n}{2n^2} = \frac{1}{2n} \to 0$$

Alternative Lösung:

$$\frac{1}{n(n+1)} + \underbrace{\frac{1}{(n+1)(n+2)}}_{\le\frac{1}{n(n+1)}} + \ldots + \underbrace{\frac{1}{(2n-2)(2n-1)}}_{\le\frac{1}{n(n+1)}} + \underbrace{\frac{1}{(2n-1)(2n)}}_{\le\frac{1}{n(n+1)}}$$

$$\le \frac{1}{n(n+1)}+\ldots+\frac{1}{n(n+1)} = (n-1)\frac{1}{n(n+1)} = \frac{\frac{1}{n}-\frac{1}{n^2}}{(1+\frac{1}{n})} \to 0$$

16j

$$a = \sqrt[3]{1-n^3} \text{ und } b = n$$

Erweitere mit

$$(a^2-ab+b^2)$$

Beachte

$$a^3+b^3 = (a+b)(a^2-ab+b^2)$$