# On $\lambda$-sets and the dual of generalized continuity*

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## 1 Introduction

The following decompositions of continuity have been recently obtained:

**Theorem 1.1** For a function $f\colon(X,\tau)\to(Y,\sigma)$ the following conditions are equivalent:

(0) $f$ is continuous.

(1) [14, Levine] $f$ is weakly continuous and weak-* continuous: improved later by Rose [19, 20].

(2) [23, Tong] $f$ is $\alpha$-continuous and $\mathcal{A}$-continuous.

(3) [24, Tong] $f$ is precontinuous and $\mathcal{B}$-continuous: this is an improvement of (2).

(4) [11, 12, Ganster and Reilly] $f$ is (sub-)LC-continuous and precontinuous: this is also an improvement of (2).

(5) [13, Ganster and Reilly] $f$ is semi-continuous (= quasi-continuous) and ic-continuous.

(6) [10, Ganster, Gressl and Reilly] $f$ is nearly continuous (= precontinuous) and weakly $\mathcal{B}$-continuous: this is an improvement of (3).

(7) [3, Chew and Tong] $f$ is weakly continuous and relatively continuous.

Several decompositions of generalized continuity can be found in [7, 8, 18, 25].

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In this paper, we try to establish a decomposition of continuity via the recently introduced notion of generalized continuity [2], i.e. we shall provide a (strictly) weaker form of continuity, which together with generalized continuity will imply continuity. Recall that a function $f\colon (X,\tau) \to (Y,\sigma)$ is called *generalized continuous* [2] if the preimage of every closed subset of $Y$ is generalized closed in $X$. A subset $A$ of a topological space $(X,\tau)$ is called *generalized closed* (= g-closed) [15] if the closure of $A$ belongs to every open superset of $A$.

We also introduce two new separation axioms. The first one is called $T_{\frac{1}{4}}$ and it is properly placed between $T_0$ and $T_{\frac{1}{2}}$ but independent from $T_D$. The other one is called weak $R_0$.

# 2 $\lambda$-sets

In 1986, Maki [16] introduced the concept of $\Lambda$-sets in topological spaces as the sets that coincide with their kernel. The kernel of a set $A$, denoted by $A^\wedge$ [16], is the intersection of all open supersets of $A$. In $T_1$-spaces every set is a $\Lambda$-set [16] and moreover it is easy to see that if every subset (or equivalently every singleton) of a topological space $(X,\tau)$ is a $\Lambda$-set, then $X$ is $T_1$. In connection of searching for the dual of generalized continuity we make the following definition:

**Definition 1** A subset $A$ of a topological space $(X,\tau)$ is called a *$\lambda$-set* (or $\lambda$-closed) if $A = L \cap F$, where $L$ is a $\Lambda$-set and $F$ is closed. Complements of $\lambda$-closed sets will be called *$\lambda$-open.*

**Lemma 2.1** *For a subset $A$ of a topological space $(X,\tau)$ the following conditions are equivalent:*

(1) *$A$ is $\lambda$-closed.*

(2) *$A = L \cap \overline{A}$, where $L$ is a $\Lambda$-set.*

(3) *$A = A^\wedge \cap \overline{A}$.* $\square$

**Lemma 2.2** (i) *Every locally closed set is $\lambda$-closed.*

(ii) *Every $\Lambda$-set is $\lambda$-closed.* $\square$

Since locally closed sets and $\Lambda$-sets are concepts, independent from each other, then a $\lambda$-closed set need not be locally closed or a $\Lambda$-set either.

**Definition 2** A subset $A$ of a space $(X, \tau)$ is called a *generalized closed set* (briefly *g-closed*) [15] if $\overline{A} \subset U$ whenever $A \subset U$ and $U$ is open.

**Lemma 2.3** *A subset $A \subseteq (X, \tau)$ is g-closed if and only if $\overline{A} \subseteq A^{\wedge}$.* $\square$

**Theorem 2.4** *For a subset A of a topological space $(X, \tau)$ the following conditions are equivalent:*

(1) *$A$ is closed.*

(2) *$A$ is g-closed and locally closed.*

(3) *$A$ is g-closed and $\lambda$-closed.*

*Proof.* (1) $\Rightarrow$ (2) Every closed set is both g-closed and locally closed.

(2) $\Rightarrow$ (3) is Lemma 2.2 (i).

(3) $\Rightarrow$ (1) $A$ is g-closed, so by Lemma 2.3 $\overline{A} \subseteq A^{\wedge}$. $A$ is $\lambda$-closed, so by Lemma 2.1 $A = A^{\wedge} \cap \overline{A}$. Hence $A = \overline{A}$, i.e. $A$ is closed. $\square$

**Theorem 2.5** *For a topological space $(X, \tau)$ the following conditions are equivalent:*

(1) *$X$ is a $T_0$-space.*

(2) *Every singleton of $X$ is $\lambda$-closed.*

*Proof.* (1) $\Rightarrow$ (2) Let $x \in X$. Since $X$ is $T_0$, then for every point $y \neq x$ there exists a set $A_y$ containing $x$ and disjoint from $\{y\}$ such that $A_y$ is either open or closed. Let $L$ be the intersection of all open sets $A_y$ and let $F$ be the intersection of all closed sets $A_y$. Clearly, $L$ is a $\Lambda$-set and $F$ is closed. Note that $\{x\} = L \cap F$. This shows that $\{x\}$ is $\lambda$-closed.

(2) $\Rightarrow$ (1) Let $x$ and $y$ be two different points of $X$. By (2) $\{x\} = L \cap F$, where $L$ is a $\Lambda$-set and $F$ is closed. If $F$ does not contain $y$, then $X \setminus F$ is an open set containing $y$ and we are done. If $F$ contains $y$, then $y \notin L$ and thus for some open set $U$, containing $x$, we have $y \notin U$. Hence $X$ is $T_0$. $\square$

In each $T_0$ non-$T_1$ space there are singletons that are $\lambda$-closed but not $\Lambda$-sets.

Recall that a topological space $(X, \tau)$ is called a $T_{\frac{1}{2}}$*-space* [15] if every generalized closed subset of $X$ is closed or equivalently if every singleton is open or closed [9].

**Theorem 2.6** *For a topological space $(X, \tau)$ the following conditions are equivalent:*

(1) *$X$ is a $T_{\frac{1}{2}}$-space.*

(2) *Every subset of $X$ is $\lambda$-closed.*

*Proof.* (1) $\Rightarrow$ (2) Let $A \subseteq X$. Let $A_1$ be the set of all open (open in $X$) singletons of $X \setminus A$ and let $A_2 = X \setminus (A \cup A_1)$. Set $F = \cap_{x \in A_1} X \setminus \{x\}$ and $L = \cap_{x \in A_2} X \setminus \{x\}$. Note that $F$ is closed and $L$ is a $\Lambda$-set. Moreover, $A = F \cap L$. Thus $A$ is $\lambda$-closed.

(2) $\Rightarrow$ (1) Let $x \in X$. Assume that $\{x\}$ is not open. Then $A = X \setminus \{x\}$ is not closed and since $A$ is $\lambda$-closed, then $A$ is a $\Lambda$-set, i.e $A = A^{\wedge}$. Since $X$ is the only superset of $A$, then $A$ is open. Hence $\{x\}$ is closed. $\square$

A topological space $(X, \tau)$ is called an $R_0$-*space* [4, 21] if for every $x \in X$ and every $U \in \tau$ containing $x$ we have $\overline{\{x\}} \subseteq U$. It is well known that a topological space is $T_1$ if and only if it is $T_0$ and $R_0$ [4]. In order to improve this result, we make the following definition:

**Definition 3** A topological space $(X, \tau)$ is called a *weak* $R_0$-*space* if every $\lambda$-closed singleton is a $\Lambda$-set.

**Theorem 2.7** *Every $R_0$-space $(X, \tau)$ is a weak $R_0$-space.*

*Proof.* Let $x \in X$ with $\{x\} = L \cap F$, where $L$ is a $\Lambda$-set and $F$ is closed. Let $y \in \{x\}^{\wedge}$ such that $y \neq x$. Clearly $y \in L$. Thus $y \notin F$ and since $X$ is $R_0$, then $\overline{\{y\}} \subseteq X \setminus F$. This shows that $x \notin \overline{\{y\}}$. Thus there exists an open set containing $x$ disjoint from $y$ and so $y \notin \{x\}^{\wedge}$. By contradiction $x$ is the only point in the kernel of $x$, i.e. $\{x\}$ is a $\Lambda$-set. $\square$

**Example 2.8** A weak $R_0$-space need not be $R_0$. Consider for example $X = \{a, b, c, d\}$ with $\tau = \{\emptyset, \{c, d\}, X\}$.

**Theorem 2.9** *For a topological space $(X, \tau)$ the following conditions are equivalent:*

(1) *$X$ is $T_1$.*

(2) *$X$ is $T_0$ and $R_0$.*

(3) *$X$ is $T_0$ and weak $R_0$.*

*Proof.* (1) $\Rightarrow$ (2) and (2) $\Rightarrow$ (3) are clear.

(3) $\Rightarrow$ (1) We need to show that every singleton is a $\Lambda$-set. Since $X$ is $T_0$, then in the notion of Theorem 2.5 every singleton is $\lambda$-closed. Since $X$ is weak $R_0$, then each singleton is a $\Lambda$-set. $\square$

Recall that a topological space $(X, \tau)$ is called $T_D$ [1] if every point is locally closed. Since every open and every closed set is locally closed, every $T_{\frac{1}{2}}$ space is $T_D$. And since a locally closed set is $\lambda$-closed, every $T_D$ space is $T_0$.

Moreover, every $T_0$-Alexandroff space is $T_D$ (an *Alexandroff space* is a space where every point has a minimal open neighborhood; this is equivalent to the fact that every intersection of open sets is again open, or with the notation of this paper, that every $\Lambda$-set is open).

To see this, if we denote by $V(x)$ the minimal open neighborhood of the point $x$, it is clear that $x \in V(x) \cap \overline{\{x\}}$, which is the intersection of an open an a closed set. If we see that there is no other point in it, we have that the point is locally closed.

Let $y \in V(x) \cap \overline{\{x\}}$, $y \neq x$. Since $y \in V(x)$, every neighborhood of $x$ contains $y$; since $y \in \overline{\{x\}}$, every neighborhood of $y$ contains $x$, but this is impossible for two distinct points in a $T_0$ space, so $V(x) \cap \overline{\{x\}} = \{x\}$, as desired.

So both $T_{\frac{1}{2}}$ and $T_0$-Alexandroff imply $T_D$. Recall that there is no relation between $T_{\frac{1}{2}}$ and $T_0$-Alexandroff spaces, since the reals with the usual topology is $T_{\frac{1}{2}}$ but is not Alexandroff; on the other hand, every finite and $T_0$ space is $T_0$-Alexandroff; however the space $X = \{a, b, c\}$ with $\tau = \{\emptyset, \{a\}, \{a, b\}, X\}$ is not $T_{\frac{1}{2}}$, since the point $\{b\}$ is neither open nor closed.

# 3 $T_{\frac{1}{4}}$-spaces

**Definition 4** A topological space $(X, \tau)$ is called $T_{\frac{1}{4}}$ if for every finite subset $F$ of $X$ and every $y \notin F$ there exists a set $A_y$ containing $F$ and disjoint from $\{y\}$ such that $A_y$ is either open or closed. Note that every $T_{\frac{1}{4}}$ space is $T_0$ (take $F = \{x\}$ with $x \neq y$).

The preceding definition leads the following characterization, which answers the following question: which spaces have all their finite sets $\lambda$-closed? The proof is very similar to that of Theorem 2.5.

**Theorem 3.1** *For a topological space $(X, \tau)$ the following conditions are equivalent:*

(1) *$X$ is a $T_{\frac{1}{4}}$-space.*

(2) *Every finite subset of $X$ is $\lambda$-closed.*

*Proof.* (1) $\Rightarrow$ (2) Let $F \subset X$ be a finite subset of $X$. Since $X$ is $T_{\frac{1}{4}}$, then for every point $y \notin F$ there exists a set $A_y$ containing $F$ and disjoint from $\{y\}$ such that $A_y$ is either open or closed. Let $L$ be the intersection of all open sets $A_y$ and let $C$ be the intersection of all closed sets $A_y$. Clearly, $L$ is a $\Lambda$-set and $C$ is closed. Note that $F = L \cap C$. This shows that $F$ is $\lambda$-closed.

(2) $\Rightarrow$ (1) Let $F$ be a finite subset of $X$ and $y$ be a point of $X \setminus F$. By (2), $F = L \cap C$, where $L$ is a $\Lambda$-set and $C$ is closed. If $C$ does not contain $y$, then $X \setminus C$ is an open set containing $y$ and we are done. If $C$ contains $y$, then $y \notin L$ and thus for some open set $U$, containing $F$, we have $y \notin U$. Hence $X$ is $T_{\frac{1}{4}}$. □

The new separation axiom $T_{\frac{1}{4}}$ is strictly placed between $T_{\frac{1}{2}}$ and $T_0$. Note also that the separation axioms "weak $R_0$" and $T_{\frac{1}{4}}$ are independent from each other.

**Example 3.2** Let $X$ be the set of non-negative integers with the topology whose open sets are those which contain 0 and have finite complement (so closed sets are the finite sets that do not contain 0). Every point is closed except 0, which is neither open nor closed nor even locally closed.

This space is neither $T_{\frac{1}{2}}$ nor $T_D$, although it is $T_0$. However it is $T_{\frac{1}{4}}$. To see this, we distinguish the following cases:

(a) $y \notin F$, where $y \neq 0$ and $0 \notin F$; then take $A_y = F$.

(b) $y \notin F$, where $y = 0$; then take $A_y = X \setminus F$.

(c) $y \notin F$, where $y \neq 0$ and $0 \in F$; then take $A_y = X \setminus \{y\}$.

On the other hand, the space $X = \{a, b, c\}$ with $\tau = \{\emptyset, \{a\}, \{a, b\}, X\}$ is $T_D$ and hence $T_0$ but not $T_{\frac{1}{4}}$, since $\{a, c\}^{\wedge} = \{a, b, c\}$ and $\overline{\{a, c\}} = \{a, b, c\}$, so $\{a, c\}$ is not $\lambda$-closed, although $\{a\}$ and $\{c\}$ are. ($\{a\}$ and $\{c\}$ are even locally closed)

So, even a finite union of $\lambda$-closed sets need not be $\lambda$-closed. However, since any intersection of $\Lambda$-sets gives a $\Lambda$-set [16], then we have:

**Theorem 3.3** *An arbitrary intersection of $\lambda$-closed sets is a $\lambda$-closed set.* $\Box$

One can ask the following question: for which spaces is the set of all $\lambda$-open subsets a topology? Call those spaces $\lambda$-*spaces*. Clearly a topological space $(X, \tau)$ is a $\lambda$-space if and only if the union of any two $\lambda$-closed sets is a $\lambda$-closed set. From Theorem 2.6 we have that every $T_{\frac{1}{2}}$ space is a $\lambda$-space. And, a $T_0$ $\lambda$-space is $T_{\frac{1}{4}}$, since from Theorem 2.5 every singleton is $\lambda$-closed and in a $\lambda$-space finite union of $\lambda$-closed sets is $\lambda$-closed. So,

$T_{\frac{1}{2}} \Rightarrow T_0$, $\lambda$-space $\Rightarrow T_{\frac{1}{4}} \Rightarrow T_0$

Note that the space $X$ given in Example 3.2 is a $\lambda$-space that is not $T_{\frac{1}{2}}$. To see this, note that a subset of $X$ is a $\Lambda$-set if and only if 0 belongs to it (the intersection of open sets contain 0; on the other hand, if $0 \in L$, put $X \setminus L = \bigcup_{i \in I} F_i$ where $F_i$ are finite sets without 0 (this is always possible) and then $L = \bigcap_{i \in I}(X \setminus F_i)$ is a $\Lambda$-set). Hence the $\lambda$-closed subsets of $X$ are precisely the closed subsets and the $\Lambda$-sets of $X$, and it is easily seen that this is a $\lambda$-space (This is an example of a space whose $\lambda$-sets are the least that are possible: only the closed sets and the $\Lambda$-sets). So:

**Question 1**: Is every $T_{\frac{1}{4}}$ space a $\lambda$-space?

Recall from [17] that a set is called a $\nu$-set if $\mathrm{int}B = B^{\nu}$ and a generalized $\nu$-set if $\mathrm{int}B \subset B^{\nu}$, where $B^{\nu}$ is the union of all closed subsets of $B$. Clearly, $A$ is a $\Lambda$-set if and only if $X \setminus A$ is a $\nu$-set, and if we recall [16] that $A$ is a generalized $\Lambda$-set if $A^{\wedge} \subset \overline{A}$, then $A$ is a generalized $\Lambda$-set if and only if $X \setminus A$ is a generalized $\nu$-set. From [17] we have that a space is $T_{\frac{1}{2}}$ if and only if every generalized $\nu$-set is a $\nu$-set or equivalently, if every generalized $\Lambda$-set is a $\Lambda$-set. On the other hand, from Theorem 2.6, a space is $T_{\frac{1}{2}}$ if and only if every set is $\lambda$-closed. If for a $T_{\frac{1}{2}}$ space $X$ every $\lambda$-closed set is a generalized $\Lambda$-set, then every set is a $\Lambda$-set, so $X$ is $T_1$. That is, in every $T_{\frac{1}{2}}$ non-$T_1$ space there is a $\lambda$-closed set that is not a generalized $\Lambda$-set. Naturally we now have the following question:

**Question 2**: What about the converse?

A space is called semi-pre-$T_{\frac{1}{2}}$ [5] if and only if every singleton is (semi)-preopen or closed.

A space is called semi-$T_{\frac{1}{2}}$ [22] if and only if every singleton is (semi)-open or semi-closed.

These two separation axioms are both weaker than $T_{\frac{1}{2}}$, as $T_D$ is.

It is not difficult to find examples of non-$T_0$ spaces that are semi-$T_{\frac{1}{2}}$ and semi-pre-$T_{\frac{1}{2}}$ respectively, so these spaces cannot be $T_D$.

On the other hand the space $X = \{a, b, c\}$ with the topology $\tau = \{\emptyset, \{a\}, \{a, b\}, X\}$ is $T_D$ and is not semi-pre-$T_{\frac{1}{2}}$ (since the point $\{b\}$ is not preopen, in fact $\mathrm{int}\overline{\{b\}} = \emptyset$). However, it is known that every $T_D$-space is semi-$T_{\frac{1}{2}}$ (note that space is semi-$T_D$ if and only if it is semi-$T_{\frac{1}{2}}$ [6]). Thus a space is $T_D$ if and only if it is semi-$T_{\frac{1}{2}}$ and every semi-closed singleton is locally closed.

We can also ask about the relations between $T_{\frac{1}{4}}$ and semi-$T_{\frac{1}{2}}$. Again, there are semi-$T_{\frac{1}{2}}$ and semi-pre-$T_{\frac{1}{2}}$ spaces that are not $T_{\frac{1}{4}}$. On the other hand, the space from Example 3.2 is $T_{\frac{1}{4}}$ but not semi-pre-$T_{\frac{1}{2}}$, since the zero point is not semi-closed. In fact, it is dense in $X$; such points are known as *generic points*.

**Question 3**: Is every $T_{\frac{1}{4}}$-space necessarily semi-pre-$T_{\frac{1}{2}}$?

# 4 The dual of generalized continuity

**Definition 5** A function $f\colon (X, \tau) \to (Y, \sigma)$ is called:

(1) *g-continuous* [2] if $f^{-1}(V)$ is g-closed in $(X, \tau)$ for every closed set $V$ of $(Y, \sigma)$,

(2) *LC-continuous* [11] if $f^{-1}(U)$ is locally closed in $(X, \tau)$ for every open set $U$ of $(Y, \sigma)$,

(3) *co-LC-continuous* if $f^{-1}(V)$ is locally closed in $(X, \tau)$ for every closed set $V$ of $(Y, \sigma)$,

(4) *$\lambda$-continuous* if $f^{-1}(V)$ is $\lambda$-closed in $(X, \tau)$ for every closed set $V$ of $(Y, \sigma)$.

Every co-LC-continuous function (in particular every continuous function) is $\lambda$-continuous but not vice versa.

**Example 4.1** Consider the classical Dirichlet function $f\colon \mathbf{R} \to \mathbf{R}$, where $\mathbf{R}$ is the real line with the usual topology:

$$f(x) = \begin{cases} 1, & x \in \mathbf{Q}, \\ 0, & \text{otherwise.} \end{cases}$$

It is easily observed that $f$ is $\lambda$-continuous. But $f$ is neither LC-continuous nor co-LC-continuous nor g-continuous, hence not continuous.

To see that g-continuity and $\lambda$-continuity are concepts totally independent from each other, consider the identity function $f\colon(\mathbf{R},\tau)\to(\mathbf{R},\sigma)$, where $\mathbf{R}$ is the real line, $\tau$ is the indiscrete topology and $\sigma$ is the usual topology. Note that $f$ is g-continuous but not $\lambda$-continuous.

Finally, we present the new decomposition of continuity:

**Theorem 4.2** *For a function $f\colon(X,\tau)\to(Y,\sigma)$ the following conditions are equivalent:*

(1) *$f$ is continuous.*

(2) *$f$ is g-continuous and co-LC-continuous.*

(3) *$f$ is g-continuous and $\lambda$-continuous.* □

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