Hyperbolic structures in stochastics, graph theory, and topology

Proposal for an FWF project by

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Contents

1. Proposal forms

The next 13 pages contain the proposal forms, namely:

- application form
- itemization of requested funding
- form sheets for international cooperation partners.

Besides Wolfgang Woess, also Agelos Georgakopoulos has contributed to the preparation of the proposal text.

2. Introduction

The two FWF projects "Asymptotic properties of random walks on graphs" (P15577, years 2002–2006) and "Random walks, random configurations, and horocyclic products" (P19115-N18, years 2006–2011) have served as the crucial "backbone" for building a research group in my areas of research at Graz University of Technology. Each of those projects was equipped with the funding for one PostDoc and one PhD student, but the group had up to 12 members, slightly varying throughout the years.

For more on this "backbone" function, please see the *Statement on the overall impact of the project* in §2.1. of the enclosed *preliminary version of the final report* of P19115-N18. This report is also available at `http://www.math.tu-graz.ac.at/~ woess/final-report-P19115.pdf`

I am again applying to the FWF for one PostDoc and one PhD position.

The natural candidate for the PostDoc position is **Dr. Agelos Georgakopoulos**, whose presence in the research group during the last two years has been very fruitful, and whose current activities make it very important to keep him in the group for a few more years and to take up his widespread interests with a very good background in graph theory.

There are several good local candidates for the PhD position: **Mr. Florian Lehner** is one of the very best mathematics students that we currently have here at TU Graz. He is already cooperating with Georgakopoulos in the context of his diploma (=master) thesis. It appears highly desirable to keep him attached to my research group. The conclusion of his master studies is going to come a bit too late for having been recruited as a DK-funded PhD student within our graduate program ("DK plus", Doktoratskolleg) *Discrete Mathematics* (see below for details). The same is true for **Mrs. Larissa Stoiser**, currently working on her master thesis on relating quasi-Monte Carlo and Markov Chain Monte Carlo methods. In any case, it appears desirable not to attach that PhD position to one specific candidate, but to carry out an international recruitment procedure. It is expected that the person who then enrolls as a doctoral student within the present project can obtain the status of an associated DK student with significant benefits from the DK program.

Compared with the previous projects, the new proposal shifts the topics a bit more towards issues regarding infinite graphs and their topology on one hand, and towards issues regarding Brownian motion and potential analysis on the other. Of course, we always maintain the philosophy of studying the *interplay* between structural properties of graphs and groups and questions related to stochastics & potential theory, with additional focus on topological issues. Indeed, it has always been a central interest of mine to do mathematics at the confluent of more than one field of specialisation.

The *hyperbolic structures* of the proposal title arise in different ways:

- Graphs which (with their discrete metric) are hyperbolic in the sense of Gromov.
- Trees as basic examples of hyperbolic graphs, used for describing the structure of more complicated spaces via *structure trees.*
- Hyperbolic spaces or graphs that are constructed from more general metric spaces, with the latter as their boundary at infinity.
- Spaces (not necessarily only graphs) that are not hyperbolic themselves, but have hyperbolic "building blocks", such as the horocyclic products of trees and the upper half plane that were studied in the preceding project.

The themes that we intend to investigate are the following.

A. Continuation of the study of Brownian motion and harmonic functions on "treebolic space" and related horocyclic products.

B. Random walks on Baumslag-Solitar groups – related with **A**.

C. Hyperbolic extensions, stochastic dynamical systems, and processes on hyperbolic boundaries.
D. A continuation of the work on Brownian motion and potential theory on metric graphs with finite total edge lengths.
E. Characterisation of the planar groups, yielding structure tree splittings of these groups; extensions to non-planar cases.
F. Using augmented trees and Gromov-hyperbolicity to describe local connectedness in topological spaces.

The last three themes are related with the presence of A. Georgakopoulos in my research group. The interplay with the first three is not obvious at first glance, but they are closely related through the stucture theory which has been one of the fundaments of my research. More details will be given below in §5.

3. FWF Doctoral program (DK-plus) "Discrete Mathematics"

This "Doktoratskolleg" (equivalent of the German "Graduiertenkolleg") was established in 2010. It is a joint project of 10 mathematicians from TU Graz and the Universities of Graz and Leoben, with substantial funding from the FWF for up to 12 years. The PhD students were internationally recruited. The salaries plus generous travel allowances for 10 of them are funded by FWF, and in addition there are up to 11 associated PhD students with salaries from different sources, but with the same travel allowances, etc. I am the speaker of this program. For more details, see `http://www.math.tugraz.at/discrete`

It is expected that the employees of the presently proposed project will interact strongly with this doctoral program, with mutual benefits.

4. Brief overview on current work and the research group

The current & recent work within my research group is documented in the enclosed *preliminary version of the final report* of FWF P19115-N18 and will not be repeated here in all details. Please see also the list of publications of the research group that is part of that report.

Currently, the group consists of the following persons. For each of them, I give a short outline of the work and some recent publications.

Four regular staff members of Institut für Mathematische Strukturtheorie

Wolfgang Woess, head of the small institute. My own scientific work in the last years has been concentrating on three themes.

One is the study of context-free languages in relation with finitely generated groups, a collaboration with *Tullio Ceccherini-Silberstein* [C-W]. This has lead to results conerning random walks on virtually free groups and on certain (tree-like) Schreier graphs of pairs (G, H), where G is a finitely generated group and H a subgroup. Among other, this has lead to a complete description of the asymptotic behaviour of random walk transition probabilities for finite range random walks on virtually free groups in [W4]. Previously, that result had been obtained for such random walks on free groups in important work of *Lalley* [L1], but the final, complete extension to virtually free groups had remained a question (asked by *Saloff-Coste*) in spite of Lalley's very interesting language-theoretic approach [L2].

Another line of research is the collaboration [P-W] with *Marc Peigné* (Tours) on random dynamical systems, that is, iteration of random i.i.d. Lipschitz mappings of a proper metric space X. Here, we are primarily interested in the critical case, where the expected value of the

logarithm of the Lipschitz constant is 0. One of our ideas is the use of a *hyperbolic extension* of X and of the mappings, a topic that may merit further investigations.

The third research theme has been Brownian motion on *strip complexes*, a collaboration with *Alexander Bendikov, Laurent Saloff-Coste and Maura Salvatori* [B-S-S-W1], [B-S-S-W2]. It started with an idea of mine for explicit description of all positive harmonic functions on *treebolic space*, and basic analytic foundations were provided mainly by Bendikov and Saloff-Coste. This subject involves current and future research and is also part of the present proposal; see below for more details.

Franz Lehner, tenured associate professor position. He is working in free probability, spectra of convolution operators, and related topics. One of the highlights of his recent research started with his joint paper with Neuhauser and myself [L-N-W] on the relation between site percolation and the spectrum of lamplighter random walks on groups: take a finitely generated group G and a finite group of "lamp states" H. Start with a symmetric probability measure μ supported on a finite set of generators S of G and consider the associated "switch-walk-switch" lamplighter random walk on the wreath product $H \wr G$, the lamplighter group. The result is that when site percolation with probability parameter $p = 1/|H|$ on the Cayley graph of (G, S) is such that all clusters are almost surely finite, then the lamplighter random walk has pure point spectrum, and one also has a precise understanding of the (finitely supported) eigenfunctions. In subsequent work [L], this was generalized to arbitrary graphs, and it was used by Lehner and Wagner [L-W] to disprove a conjecture of Atiyah. In other prominent recent work, Lehner together with Belinschi, Bozejko and Speicher [B-B-L-S] showed that the normal distribution is freely infinitely divisible.

Lorenz Gilch, assistant position (higher level = with PhD), returned on March 1, 2011 from a semester's stay at University of Geneva (group of A. Karlsson). He is working on random walk on free products and trees, entropy, rate of escape and other. Among his recent work, I like very much the careful study, together with E. Candellero [C-G], of the asymptotic behaviour of return probabilities of random walk on free products of groups, in particular, integer lattices. This takes up an old example of Cartwright and elaborates in full detail the phase transitions (with up to three phases) that can occur in this class of random walks. Before that, together with S. Müller (a previous PostDoc in the group) in [G-M], he has obtained interesting results about random processes on trees which are directed covers of (finite or infinite) graphs, not only generalizing older results of Nagnibeda and myself [N-W], but also adding new insights such as a method for calculating the asymptotic entropy and results applying to random walk in random environment. In fact, Gilch's note on formulae for types of rates of escape of random walks [G] anticipates the recent result of Ledrappier [Ld] on analyticity of the rate of escape (and, implicitly, also entropy) on virtually free groups.

Christoph Temmel, assistant position (lower level = working towards PhD), joint PhD student of Prof. Pierre Mathieu (Marseille) and myself within a "Cotutelle"-project, working primarily on k-dependent percolation, with substantial input from Mathieu. Temmel has just submitted a first paper [M-T], joint with Mathieu, on the range of critcality of k-dependent percolation processes on trees. A second paper is almost finished, with an even stronger emphasis on themes from statistical mechanics. Temmel is a very determined young researcher.

In addition, there are **four project collaborators:**

Agelos Georgakopoulos, FWF funded PostDoc until 31 March 2011 (thereafter in Ottawa with V. A. Kaimanovich for a few months). He comes from the Hamburg school of (infinite) Graph Theory of *Reinhard Diestel.* Main topics there have been topological considerations regarding infinite graphs and their end space, translating classical results on finite graphs to infinite ones. For example, in this theory, cycles in a finite graph are replaced with topological

cycles in the end compactification of a locally finite graph. Agelos has contributed in an impressive way to that work, see e.g. [G2]. He is also very open towards various other topics, such as Gromov hyperbolic graphs, harmonic functions with finite power ("energy") on infinite graphs [G4], the classification of planar Cayley graphs [G6], and Brownian motion on metric graphs. His research interests have influenced the present proposal substantially.

Ecaterina Sava obtained her PhD in december 2010. Romanian, she joined the group several years ago as a PhD student funded by the NAWI Graz cooperation project in the natural sciences of the two Graz universities. She has worked primarily on Poisson boundaries of lamplighter random walks [S], and has also collaborated with another PhD student in the group, **Wilfried Huss**, who also received his PhD at the end of 2010 and is now a PostDoc in Siegen (Germany) with *Uta Freiberg.* The topic of the collaboration of Huss and Sava is internal aggregation on the comb lattice, such as the divisible sandpile, rotor router aggregation and internal diffusion limited aggregation. They are preparing two papers on these subjects. Sava now has a PostDoc position as the coordinator of the DK-plus "Discrete Mathematics" mentioned in §3 above.

Elisabetta Candellero is a PhD student with a project grant of the Austrian Academy of Sciences ("Doc fForte"). She came to Graz as an Erasmus exchange student from Torino and elaborated her master thesis under my supervision, after which she started working towards PhD here in Graz. She has collaborated a lot with Lorenz Gilch on various questions related with random walks on free products, see above. Her current interest is in the contact process on free products of groups. The central questions here are in which way properties of the respective process on the factors determine the behaviour on the free product.

Tetiana Boiko is a "recent acquisition". She is an associated PhD student within the DK "Discrete Mathematics", see §3. Boiko is funded via the gender program of TU Graz. She did her master in mathematics in Kharkov in Ucraine, on an interesting theme of potential theory regarding properties of the Riesz measure for subharmonic functions in the open unit disk, resp. upper half plane. Herr current task is to understand what those results – which were formulated in the classical Euclidean setting – mean in terms of hyperbolic geometry, and then to understand their analogues in the discrete setting of trees and nearest neighbour transition operators. Note how well this fits into the general themes of the present proposal, with which Boiko's work is of course closely related, even though her funding comes from another source.

Regarding the important connection with the DK "Discrete mathematics", Franz Lehner (not to confound with Florian Lehnar !) figures as an associated scientist with the DK, while Tetiana Boiko and Elisabetta Candellero are associated DK students.

Johannes Cuno from Frankfurt (where he has written his master thesis under the supervision of Robert Bieri) will join the group in May 2011 as a PhD student, funded by a regular PhD position within the DK-plus "Discrete Mathematics".

Remark: at the time of preparation of this proposal, Georgakopoulos was still in Graz, and Cuno not yet here. At the time of refereeing, Georgakopoulos will already be in Ottawa, while Cuno will have joined the group.

5. Project research: details

A. Brownian motion and harmonic functions on "treebolic space" and related horocyclic products.

This is a continuation of the work begun by Bendikov, Saloff-Coste, Salvatori and Woess [B-S-S-W1], [B-S-S-W2]. The paper [B-S-S-W1] lays down the rigorous foundations for the construction, and regularity results, of the Laplacian, heat kernel and Brownian motion on *strip*

complexes. A strip complex is a special type of Riemannian complex obtained by gluing "strips" (manifolds) along their natural boundaries according to a given graph structure. This builds on the theory of local Dirichlet forms, see e.g. Fukushima, Oshima and Takeda [F-O-T] and Sturm [St1], [St2],[St3]. In [B-S-S-W1], a more concrete description of the resutling Laplacians is provided as the closures of operators that are classical second order elliptic differential operators in the smooth part of the complex and whose domains of definition involve Kirchhoff type laws along the singular parts of the complex, the *bifurcation manifolds.*

Previously, Brin and Kifer [B-K] introduced Brownian motion on 2-dimensional Euclidean complexes, while the Dirichlet form approach on more general Riemannian complexes was discussed by Eells and Fuglede [E-F], without giving the type of regularity results provided for strip complexes in [B-S-S-W1].

Treebolic space (the name was coined by Saloff-Coste) is a Riemannian complex obtained by glueing together pieces (strips) of hyperbolic upper half plane in a tree-like fashion. One can also say that it is the horocyclic product of a homogeneous tree and hyperbolic plane. It appears, for example, as a space on which the amenable Baumslag solitar group $BS(q,1)$ acts with compact quotient, see e.g. Farb and Mosher [F-M]. It is also implicit in older work of Kaimanovich [K1]. One of the interesting facts is that the geometry of this space as well as the other issues addressed here (Brownian motion and harmonic functions) have many common aspects with the lamplighter group $\mathbb{Z}_q \wr \mathbb{Z}$ (and random walks on that group) as well as with the so-called Sol-groups (and Brownian motion). The fact that the lamplighter group can be described as the horocyclic product of two trees was used extensively in previous projects in collaboration of Woess with Brofferio and with Bartholdi [W3], [Br-W1], [Br-W2], [Ba-W]. More recently, Brofferio, Salvatori and Woess [B-S-W] have begun a similar study of $\mathsf{Sol}(p,q)$, the group of all matrices

$$\begin{pmatrix} e^{pz} & x & 0 \\ 0 & 1 & 0 \\ 0 & y & e^{-qz} \end{pmatrix}, \quad x,y,z \in \mathbb{R},$$

where $p,q>0$. [B-S-W] make extensive use of the fact that this Lie group has the geometry of the horocyclic product of two hyperbolic planes. This material merits a continuation in further, very interesting directions. In particular, one can define an analagous product of a tree with hyperbolic plane, where the Busemann function with respect to a single boundary point of the tree is replaced with a different natural level function (by orienting the tree's edges such that each vertex has r incoming and s outgoing edges). One obtains another strip complex, on which the nonamenable Baumslag-Solitar group acts, see **B** below. There are challenging additional difficulties (not apparent at first sight) when one studies Brownian motion and potential theory of that space.

In order to complete the bibliographical picture, one should mention here that the horocyclic product structure of $\mathbb{Z}_q \wr \mathbb{Z}$ and $\mathsf{Sol}(p,q)$ appears in groundbreaking work of Eskin, Fisher and Whyte [E-F-W1], [E-F-W2] on quasi-isometry types of those classes of groups.

B. Random walks on Baumslag-Solitar grous.

The above outline should make it clear that **B** is closely linked with **A**. It is a challenge that "persecutes" me since several years, and was already briefly indicated as a long term aim in the previous project proposal, to describe the Martin boundary for simple and other random walks on $BS(r,1) = \langle a,b \mid ab = b^r a\rangle$ as well as on treebolic space, similarly to the lamplighter group as in [Br-W1].

It may be of interest to note here that $BS(2,1)$ has an important appearance in wavelet theory.

One would expect, as for the lamplighter group, that the Martin compactification of Sol as well as of treebolic space is obtained as the closure of the respective horocyclic product in the product space of natural geometric compactifications of the factor spaces of that product.

In particular, treebolic space should serve as an indication of how one can proceed on its cocompact lattice $BS(r,1)$. Analogous, but harder questions concern the non-amenable Baumslag-Solitar groups $BS(r,s) = \langle a, b \mid ab^r = b^s a\rangle$.

The Poisson boundary (i.e. the study of bounded harmonic functions instead of the significantly harder task of determining the positive ones in terms of the Martin boundary) of $BS(q,1)$ is relatively easy to understand by looking at treebolic space, which even was not yet "invented" when Kaimanovich [K1] was able to describe that boundary. The generalized version of treebolic space described in **A** will be useful for finding the Poisson boundary of $BS(r,s)$ in general, and as mentioned above, there are additional "hidden" difficulties here which arise when the level projection of the random walk (which is a classical random walk on the integers) has drift 0. Namely, when the drift is non-zero, one can expect the boundary to be determined via Kaimanovich's [K2] powerful geometric *strip criterion*. However, when the drift is 0, the boundary will not be trivial (the group is non-amenable !) and new ideas have to be developed.

C. Hyperbolic extensions, stochastic dynamical systems, and processes on hyperbolic boundaries.

At first glance, the "dynamical systems" part of this topic may not appear to be linked with the other topics outlined above. However, a "hyperbolic upper half plane" situation is present here, and very useful. Peigné and Woess [P-W] have taken up the study of processes $X_n^x = F_n \circ F_{n-1} \circ F_1(x)$ taking place on a proper metric space X, where the F_n are i.i.d. random Lipschitz mappings $\mathsf{X} \to \mathsf{X}$. The most interesting case here is the critical one, when the logarithm of the (random) Lipschitz constant of F_n has 0 expectation. Appealing to methods developed by Benda [B], we deduce new and complete results regarding existence and uniqueness of an invariant Radon measure for the process, as well as ergodicity. A basic tool here is the introduction of a hyperbolic extension $\widehat{\mathsf{X}}$ of X. This is a hyperbolic metric space similar to upper half plane, whose lower part of the boundary is X, and there is one additional boundary point at infinity. Lipschitz mappings extend naturally to mappings with Lipschitz constant 1 that resemble affine transformations.

I believe that this idea merits to be pursued further, and that one should use it to undertake a detailed study of classes of key examples such as the reflected–affine recursion touched briefly in [P-W].

It is also notworthy that the construction of this hyperbolic extension bears some strong similarities (though being quite different in its details) with the way how Kaimanovich [K3] associates a hyperbolic graph with a given PCF fractal. This graph is obtained by adding "horizontal" edges to a regular, rooted tree (the root may also sit at infinity) and has the fractal as its hyperbolic boundary. This provides a link between stochastic processes on the fractal and random walks on that graph.

A side remark, also challenging for future (not necessarily immediate) research: in recent, impressive work, Kigami [Ki] has used the fact that the Cantor set arises naturally as the boundary of a tree for constructing a class of natural stochastic processes on that set via random walks on the tree. This approach appears to have a natural extension to hyperbolic graphs (or groups) and their boundary. As we have seen above, PCF fractals appear as such boundaries, and the nature of the resulting processes should be investigated and related with known processes on fractals.

D. Brownian motion and potential theory on metric graphs with finite total edge lengths.

Recall that a metric graph is a 1-complex, the 1-skeleton of a (finite or infinite) graph, where each edge is considered as an interval of a given positive edge length, and those edges are glued together at the vertices. The Laplacian and Brownian motion on metric graphs are well understood when those edge lengths are bounded away from 0. Informally, inside each open edge, one takes the ordinary one-dimensional Laplacian, and the domain is such that a Kirchhoff-type condition has to hold for one-sided derivatives along the incoming edges at each vertex. One might even interpret this as the simplest variant of a strip complex as in [B-S-S-W1] and apply what is elaborated there, but the construction is not as hard as in that general framework.

When the edge lengths are not bounded below, in particular, when the graph is infinite and the total sum of the edge lengths is finite, the rigorous construction is more complicated. In that case, Brownian motion should be able to reach the boundary (i.e., the space of ends of the graph) in finite time, and one needs a construction where the process continues to evolve after that as well.

Regarding graphs with these properties, *Agelos Georgakopoulos* is an expert. He and *Konrad Kolesko,* a very clever PhD student from Wroclaw who was employed for three months in the final phase of the previous FWF project P19115, have started research on Brownian motion on such metric graphs. This is very promising, but only in its initial phase and should definitely be continued within the project proposed here. Hyperbolic structures, in particular infinite trees, come up as the first class of test cases.

Let me also mention here the recently started collaboration of Georgakopoulos with *Vadim A. Kaimanovich* on the relation of harmonic functions with finite Dirichlet sum on infinite networks with Martin boundary theory, a research that is going to be pursued during Georgakopoulos' stay in Ottawa after the termination of project FWF P19115-N18 and will quite likely be still ongoing in 2012.

E. Characterisation of planar groups and structure tree splittings.

This is, at least at first glance, quite different from the themes outlined so far. Nevertheless, structure trees and structure tree splittings have played an important role in part of my work, in relation with boundary theory as well as transition probability asymptotics, see e.g. [W1] and [W4].

The study of groups that have Cayley graphs embeddable in the Euclidean plane, called *planar groups*, has a tradition starting in 1896 with Maschke's characterization of the finite ones. Among the infinite planar groups, those that admit a *planar Cayley complex*, i.e., a Cayley complex embeddable in the plane, have received a lot of attention. These groups are now well understood due to the work of Macbeath [M], Wilkie [Wi], and others; see [Z-V-C] for a survey. They are closely related to the *surface groups* [Z-V-C, Section 4.10], that is, fundamental groups of surfaces. The latter have had an important appearance in the spectral theory of random walks, see the outline and references in [W2, p. 138], with interesting open problems regarding the nature of the spectral radius and resolvent.

There are planar groups that have no planar Cayley complex. These groups are the subject of on-going research and they are not yet completely understood. For example, it is an open problem of Droms et. al. [D-S-S] whether these groups admit an effective enumeration.

Important recent contributions to this field are due to Georgakopoulos. On more than 100 pages [G5], [G6], he has achieved a complete description of all the cubic Cayley graphs of planar groups. This work solves several open problems, and it also motivates many new ones, some of which to be attacked in the proposed project:

- Classify all planar Cayley graphs and prove Droms' conjecture that planar groups admit an effective enumeration.
- Obtain a graph theoretical version of Stallings' theorem, settling Mohar's tree amalgamation conjecture [Mo].
- Study spectral properties of planar Cayley graphs.
- Study Hamilton circles in planar Cayley graphs.

The last of those questions appears to be ideally suitable for the interests of the possible future PhD student Florian Lehner. The Hamilton circles here are topological cycles in the end compactification, compare with the significant paper of Georgakopoulos [G2] on the infinite version of Fleischner's theorem.

F. Gromov-hyperbolicity and local connectedness in topological spaces.

Gromov [Gr] has shown that for every compact metric space X there is a locally finite hyperbolic graph (i.e., its discrete metric is Gromov-hyperbolic) whose hyperbolic boundary is isometric with X.

Georgakopoulos (work in preparation) has exploited this fact to show that the question of whether a metric space is path-connected can be reduced to a question about a graph. He applies this to yield a graph-theoretical proof of the well-known theorem of Hahn and Mazurkiewicz. The approach can be summarised as follows.

It was proved in [Gr, Corollary 7.2.M]) that if a graph G is hyperbolic then there is an assignment of positive lengths $e \mapsto \ell(e)$ to the edges e of G such that the completion $|G|_\ell$ of the resulting metric graph yields the hyperbolic compactification of G. This general type of completion was studied by Georgakopoulos in [G3], who obtains that the graph can be then decomposed into a rooted tree T, which is a geodetic spanning tree of G, and an infinite sequence L_i of finite graphs so that $G = T \cup \bigcup L_i$ and $V(L_i)$ is the set of vertices of T in its ith level. Thus G is an augmented tree in the sense of Kaimanovich [K3]. (Note the link with topic **C** !)

The ends of T can be canonically mapped to points of X, and the structure of the level graphs L_i is determined by the topology of X.

Then, one reduces the question of whether X is path-connected or locally connected to a question about the existence of certain topological paths in the end compactification of T. The latter question is answered exploiting methods similar to those of [G1].

The plan is to apply this approach to obtain new results concerning the connectedness properties of compact metric spaces. Open problems on which our approach can be tested are the following.

(a) Let X be a nondegenerate hereditarily equivalent continuum. Must X be (homeomorphic to) either an arc or a pseudo-arc ? The history of this problem goes back to Mazurkiewicz (1921).

(b) Is the Mandelbrot set locally connected ? This is of course a famous and difficult problem.

Note here that the hyperbolic extension outlined in **C** is a "sister structure" of the constructions of Gromov and of Kaimanovich. This clarifies once more that while at first glance, some of the topics outlined above appear to be quite different, they are strongly linked by a common way of thinking.

As stated in my previous project proposal, the research topics outlined here may be slightly more than a group of two or three young researchers collaborating with me may be able to solve completely within three years. But again, the above is a reservoir of interesting questions, and already in the past project, practically all issues raised in the proposal were addressed successfully by members of my research group & myself during the last years: once more, the present project should have a significant impact on the research group on the whole, going beyond the collaborators which are funded directly by the project.

6. Project duration

This project should start in late 2011 or early 2012 and have a duration of three years.

7. Project personnel

I am applying (again) for one PostDoc and one PhD position. As mentioned already in the introduction, the primary candidates are **Dr. Agelos Georgakopoulos** (PostDoc) and **Mr. Florian Lehner** or **Mrs. Larissa Stoiser**, respectively (PhD).

The main educational aspect of the present project is its tight link with the FFW doctoral program (DK-plus) "Discrete Mathematics", of which I am the speaker.

Note that the topics presented here have a strong intersection with, but go beyond the ordinary scope of discrete mathematics alone. This reflects my ongoing interest to do research at the intersection of different mathematical disciplines.

8. International collaboration

As in the precending project, my multiple international contacts will have an ongoing benefit for the research group, and in particular, for the project staff.

As in previous years, there are several ways of funding for inviting foreign partners to Graz for stays of one week to 2 months:

(1) visiting professorships at TU Graz (with not too attractive payment, but sufficient to cover costs);

(2) very few short visits in Graz are funded by the budget for colloquium speakers ("Gastvortragende") of the faculty at TU Graz;

(3) there is the possibiilty of exchange of teaching personnel within the EU Erasmus programme;

(4) in the recently established European Science Foundation programme "RGLIS" (of which I am a member of the steering committee), funding for short visits is available.

Again, as before, within the present project, I also propose to establish specific international collaborations on the basis of visits of a few weeks in either direction (guests coming to Graz, or project collaborators from Graz going abroad). This aims at an exchange related to the specific topics of the present project.

I should add here that already in the past, my strategy always was to use all possible funding sources optimally. In particular, the primary use of the project funding always has to be the funding for the project personnel, while only very rarely did I use project money for project related conference participations of myself.

The collaboration partners and related requested funding are of course specifically intended for the research proposed in this project. In particular, the following persons from abroad should be part of an exchange within the work on the topics proposed above.

(A) Alexander Bendikov, Professor at Wroclaw University, is a very reliable expert in potential theory and construction of diffusion operators, with a recently increasing interest in random walk questions. The inclusion of his name in this list is related with the recent shift of parts of my research interests towards structures such as treebolic space and related complexes, and the study of Laplacians and Brownian motion on them. The interaction with Wroclaw University is not only limited to him, but also includes the group of **Ewa Damek** and her younger colleague **Dariusz Buracewski**. Visits of my group members to Wroclaw will be profitable in the context of more than one of the topics outlined above. Bendikov is already scheduled to come to Graz as a TU-funded visiting professor in autumn, 2011, and later exchange visits should follow.

(B) Daniel Lenz at Jena University (Germany) is a younger professor whose research work is covering a range of topics that is getting closer and closer to those of my research group. He is primarily an anlalyst, strong in spectral theory of metric graphs, their Laplacians, Schrödinger operators, and related objects. The "catalyst" for establishing the contact with Lenz has been **Florian Sobieczky,** who – after several years in Graz and then in Jena – is now moving to the US. Lenz has very talented young collaborators, such as **Matthias Keller**, working for example on planar graphs from an analytical viewpoint [Ke]. Good contacts of my young people with this group may have a fruitful impact on both sides.

(C) Bojan Mohar is a longstanding acquaintance of mine through our common interest in spectral theory of infinite graphs. Our old survey [M-W] on this topic is still cited frequently. For many years in Ljubljana (Slovenia), he is now at simon Fraser University (Vancouver), but often back to Ljubljana. He is a leading expert in graph embeddings on surfaces, and in that context as well as related issues, his collaboration with Agelos Georgakopoulos will be an essential part of the project.

(D) Vadim A. Kaimanovich is since many years a partner of the research activites of myself and my group of younger collaborators. A year ago, he moved from Bremen to the University of Ottawa, Canada. He is the leading expert on Poisson boundary theory and related subjects. His recent work comprises, among other, a collaboration with Sobieczky on horocyclic products, and a very promising collaboration with Georgakopoulos on Dirichlet harmonic functions. The continuation of that collaboration will be part of the proposed project. Already after the end of the previous project FWF P19115-N18, Georgakopoulos is now spending a few months at Ottawa, with funding provided by Kaimanovich. After returning to Graz, Georgakopoulos should of course continue the collabaration.

(E) Finally, **Laurent Saloff-Coste,** professor at Cornell University, is a leading figure in current research on random walks on groups, finite Markov chains as well as analysis and potential theory on manifolds, groups and other structures. He has been member of the editorial boards of several top journals, and he is part of the advisory board of the DK "Discrete Mathematics" mentioned above. We have several joint papers, already published as well as in preparation. Saloff-Coste has come to TU Graz as a visiting professor in 2003. Since now he also is the chairman of the Mathematics Department at Cornell University, it has become harder to get him to Graz, but sending my collaborators to Cornell is highly profitable for them. Several of them have taken part, or will take part in the excellent Cornell summer school in probability, which so far was not directly organised by Saloff-Coste, but has offered them a good opportunity to interact with him. Further exchange is of course planned.

9. Research plan

In mathematical research it is not very common to follow a fixed timetable such as, e.g., in laboratory experiments. Of course, there is a certain logical order for proceeding.

In the first two years, the postdoc research assistant (i.e., A. Georgakopoulos) should conclude his work with K. Kolesko (Wroclaw) on Brownian motion on metric graphs with finite total edge lengths (topic **D**). Input of A. Bendikov may be useful in this context. Exchange visits Graz $\leftrightarrow$ Wroclaw of Georgakopoulos and Kolesko (respectively) have not been included in the below funding plan, because it is expected that this can be paid from other sources. In the same period, Georgakopoulos can conlcude his collaboration with Kaimanovich and carry ahead his work on planar groups (topic **E**), maintaining contact with B. Mohar. The work on hyperbolicity and local connectedness (topic **F**) could rather be done in the final year.

Stochastic dynamical systems (topic **C**) as well as parts of the more graph theory oriented topics **E** and also **F** would be very well suited for a new PhD student; the choice should depend on

the specific abilities and preferences of the chosen one among the candidates for the PhD position. **E** and **F** should rely on a strong interaction of the PhD student with A. Georgakopoulos besides myself. The timetable for the work of the PhD student will follow the logical path, starting with getting thouroughly acquainted with the subject in the first year. The first 3 semesters should also have a focus on fulfillment of the curricular requirements (=courses and seminars) of PhD studies, while in the 2nd and 3rd year the PhD student should get used to attending conferences and – as soon as possible – summer schools and related activities where she/he can set foot in international mathematical community. Towards the middle or end of the 2nd year one may hope for a first publication.

The research on the other topics, **A** and **B**, should be carried out within the whole group, strongly including my own input and participation.

10. Importance, impact, dissemintation

Regarding the impact, please see once more the *Statement on the overall impact of the project* in §2.1 of the enclosed *preliminary version of the final report* of P19115-N18. The proposed project will be the backbone of the research group at Institut für Mathematische Strukturtheorie of TU Graz to a much larger extent than the sole employment of a PostDoc and a PhD student. It will allow this group to maintain its good reputation, which we managed to acquire in the last 8 years, owing greatly to the preceding two FWF projects.

Tightly linked with the doctoral program (DK-plus) "Discrete Mathematics", it will be an important part of the good standing and visibility of Mathematics in the universities located in the Austrain region of Styria.

The educational aspect can also be seen in relation with the so-called MINT-initiative of the Austrian Ministry of Science and Research, which aims at bringing more good students to the fields of Mathematics, Natural Sciences and Technology, and in relation with the efforts of the Austrian Mathematical Society (of which I am the chairman of the regional group in Styria) to establish an Austrian Year of Mathematics in 2012.

Dissemination of project results will follow the well-established and efficient habits of the international mathematical comunity. Postings on ArXiV usually reach the target audience very effectively, well before papers appear in a peer-reviewed journal. Conference talks and poster presentations are an obvious part of our acitivities. Support by the FWF is of course mentioned on all those occasions.

Maintenance of webpages (of the institute as well as individual ones) complete the picture.

11. Requested funding

The requested funding is primarily for a Post Doc fellowship and for a Ph.D. fellowship, each for three years.

The team members should get opportunities to visit the abovementioned international partners at the universities of Jena , Ljubljana (during one of the regular stays of Mohar in his home country), Wroclaw and Ottawa, and in particular Cornell. Here, I am estimating the costs for an overseas flight at 800 EURO and (as below) the daily costs during the stay at 75 EURO. Travel costs within Europe are estimated at 370 Euro, except for Ljublana, where 100 EURO should suffice.

Again, I also plan to bring those partners to Graz, using project funds for periods of 8–12 days for three of them. A visit of Kaimanovich from overseas should take place on an occasion when he is already in Europe. Additional stays should be financed by other sources, following my abovementioned principle to use all available funding possibilities optimally. Further exchange

visits, e.g. involving colleagues from Hamburg (Reinhard Diestel), Tours (M. Peigné) or Marseille (Ch. Pittet, P. Mathieu) should use different sources.

The funding of the vistiors' stays is computed by taking the sum of the room charge for the most convenient, low-prized nearby hotel ("Pension Johannes"), which currently charges 47 Euro per night (for TU Graz institutes), plus the official daily allowance of 26,40 Euro according to the "Reisegebührenvorschrift des Bundes" (federal reimbursement rule), amounting to a total of 73,40 Euro per day. I am estimating their average travel costs only within Europe at an amount of approximately 370 EURO. Thus, a visit of 10 days amounts to estimated costs of 1.170 EURO.

	1st year	2nd year	3rd year
PostDoc salary	58.780,–	58.780,–	58.780,–
PhD salary	33.620,–	33.620,–	33.620,–
Bendikov to Graz		1.170,–	
Kaimanovich to Graz	1.170,–		
Lenz to Graz		1.170,–	
PostDoc to Ottawa		1.700,–	1.700,–
PostDoc to Ljubljana	850,–		
PhD to Cornell	1.700,–		1.700,–
total (Euro)	96.120,–	96.440,–	95.800,–

Grand total: 288.360,– Euro

Wolfgang Woess, Graz, March–April 2011.

12. References

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[B] Benda, M.: *Schwach kontraktive dynamische Systeme,* Ph. D. Thesis, Ludwig-Maximilans-Universität München (1998).

[B-S-S-W1] Bendikov, A., Saloff-Coste, L., Salvatori, M., and Woess, W.: *The heat semigroup and Brownian motion on strip complexes,* Advances in Mathematics **226** (2011) 992–1055.

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[C-G] Candellero, E., and Gilch, L.: *Phase transitions for random walk asymptotics on free products of groups,* Random Structures and Algorithms, in print.

[C-W] Ceccherini-Silberstein, T., and Woess, W.: *Context-free pairs of groups. I - Context-free pairs and graphs*, preprint (2009).

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[L-W] Lehner, F., and Wagner, S.: *Free lamplighter groups and a question of Atiyah,* preprint, 2010.

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[St2] Sturm, K-T.: *Analysis on local Dirichlet spaces. II. Upper Gaussian estimates for the fundamental solutions of parabolic equations.* Osaka J. Math. **32** (1995), pp. 275–312.

[St3] Sturm, K-T.: *Analysis on local Dirichlet spaces. III. The parabolic Harnack inequality.* J. Math. Pures Appl. **75** (1996), pp. 273–297.

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[W2] Woess, W.: *Random Walks on Infinite Graphs and Groups*, Cambridge Tracts in Mathematics **138**, Cambridge University Press, Cambridge, 2000.

[W3] Woess, W.: *Lamplighters, Diestel-Leader graphs, random walks, and harmonic functions,* Combinatorics, Probability & Computing **14** (2005) 415–433.

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[Z-V-C] Zieschang, H., Vogt, E., and Coldewey, H.-D.: *Surfaces and Planar Discontinuous Groups.* Revised and expanded translation from the German by John Stillwell. Lecture Notes in Mathematics **835**, Springer, Berlin, 1980.

13. Curriculum vitae and publications of Wolfgang Woess

1) Education and employment

Born in Vienna (Austria) 23 July 1954.

Studies of Mathematics at the Technical University of Vienna and the Universities of Munich and Salzburg.

1978: Diploma (≡ Master) degree in Mathematics at the Technical University of Vienna.

1980: Ph. D. in Matematics at the University of Salzburg (supervisor: Peter Gerl).

1980–81: Research grant at the University of Salzburg.

1982–88: University assistant (Assistant professor) at the Institute of Mathematics and Applied Geometry of the Montanuniversität Leoben (Austria).

1984–85: on leave, research fellowship (sponsored by the Italian CNR) at the Department of Mathematics of the University of Rome (research group of A. Figà-Talamanca).

1985: "Habilitation" in Mathematics at the University of Salzburg.

1987: Biennial prize (Förderungspreis) of the Austrian Mathematical Society.

1988–1994: Associate professor for Mathematical Analysis at the University of Milan (Italy).

1994–1999: Full professor of Probability and Statistics at the University of Milan (Italy) and (1998–1999) at the second University of Milan.

Since September 1999: Full professor of Mathematics at the Technical University of Graz.

2) Organizational duties and activities

1990–1999: Associate Editor of the journal "Circuits, Systems and Signal Processing"

2000–2003: secretary of the Austrian Mathematical Society (ÖMG)

2005– : Head of the committee for doctoral studies at TU Graz.

2007– : Member of the Senate of TU Graz.

2009– : Chairman of the regional section of Styria of the Austrian Mathematical Society (ÖMG).

3) Visiting professorships

April 1986: Visiting professor at the Universities of Rome and Milan.

Spring 1987: Visits at several universities in the U.S. (Washington University - St. Louis, City University of New York) and Canada (Université de Montréal, Simon Fraser University).

Summer 1989: Visiting professor at the University of Sydney (Australia).

May/June 1991: Visiting professor at the University of Salzburg (Austria).

April–May 1992: Visits at several universities in the U.S. (Harvard Univ., CUNY, Washington Univ. - St. Louis, USC and UCLA).

May 1993: Visiting professor at Université Paris 7.

April/May 1995: Visiting professor at the University of Salzburg (Austria).

May 1996: Visiting professor at Université de Rennes-I.

April/May 1997: Visiting professor at the University of Vienna.

May 1998: Visiting professor at the Technical University of Graz.

Fall semester 1998/99: on leave from Milano, visiting professor at the Universities of Vienna and Linz.

August/September 2000: Visiting professor at Cornell University.

April/June 2002: Visiting professor at Insitut H. Poincaré, Centre Emile Borel.

April 2004: Visiting professor at the University of Sydney.

April 2005: Visiting professor at the University of Rome-I.

August 2005: Professor of the probability course at the International Summer University in Perugia.

May/June 2006: Visiting professor at the University of Tours.

July/August 2007: Visiting professor at the Kyoto University and Tohoku University.

4) Organization of conferences

Organization (together with V. A. Kaimanovich, M. A. Picardello and L. Saloff-Coste) of the INdAM conference "Random Walks and Discrete Potential Theory" (June 1997, Cortona, Italy).

Organization of the special semester "2001 - Random Walks" at the Schrödinger Insitute in Vienna (together with V. A. Kaimanovich and K. Schmidt) and of the conference "2001 - Fractals in Graz" (June 2001, together with P. Grabner).

Member of the program committee of the congress of the Austrian Mathematical Society in collaboration with UMI and SIMAI, (Bozen/Bolzano, September 2003).

Organization of the conference "Geometric Group Theory, Random Walks, and Harmonic Analysis" (June 2004, Cortona, Italy).

Special session "Stochastic Analysis on Metric Spaces", (organized together with L. Saloff-Coste and K-Th. Sturm), Joint meeting of AMS, DMV and ÖMG (2005, Mainz, Germany).

RDSES/ESI Educational Workshop on Discrete Probability at the Erwin Schrödinger Institute (organized with V.A. Kaimanovich and K. Schmidt; 2006, Vienna).

Workshop on "Boundaries" within the EU Curie program "GROUPS: European training courses and conferences in group theory" (organized with W. Huss and E. Sava, 2009, TU Graz).

Workshop "Analytic enumeration methods in combinatorics, probability and number theory" (organized with W. Huss and E. Sava, 2010, TU Graz).

Spring school "Discrete probability, ergodic theory and combinatorics", Courses by Geoffrey Grimmett, Jeff Steif, Anders Karlsson, Michael Björklund (organized with E. Sava, 2011, TU Graz).

5) Externally funded projects

The following externally funded projects were carried out at Institut für Mathematische Strukturtheorie since 2001.

A. Spectral Problems and Noncommutative Probability Theory

FWF (Austrian Science Fund) Schrödinger Return Project Nr. R2Mat

1.7.2001 – 30.11.2004[1]

Project coordinator: Prof. Wolfgang Woess

Fellow: Dr. Franz Lehner (now associate professor at the same institute)

[1]Dates are in the format day.month.year

B. Asymptotic Properties of Random Walks on Graphs

FWF (Austrian Science Fund) Projekt Nr. P15577-N05

1.10.2002 – 15.7.2006

Project coordinator: Prof. Wolfgang Woess

Project personnel: Dr. Sara Brofferio (1.10.2002 – 28.2.2003),
Dr. Ronald Ortner (1.10.2002 – 31.7.2003, now at Montanuniversität Leoben),
Dr. Markus Neuhauser (1.10.2003 – 30.9.2005, then at Universität Göttingen, now Vienna),
Dr. Florian Sobieczky (1.10.2004 – 31.8.2005 and 1.1. – 30.4.2006),
Dr. Elmar Teufl (1.4.2005 – 30.9.2005, now at Universität Bielefeld)
Dr. Adam Timar (15.1. – 15.7.2006, now at University of British Columbia, Vancouver)

External collaborators: Prof. Laurent Bartholdi (EPFL Lausanne),
Prof. Vadim A. Kaimanovich (Université de Rennes),
Prof. Laurent Saloff-Coste (Cornell University),
Prof. Andrzej Żuk (ENS Lyon)

C. Internal Diffusion Limited Aggregation on Non-Homogeneous Structures

EU Marie Curie PostDoc fellowship HPMF-CT-2002-02137

1.3.2003 – 31.8.2004

Project coordinator: Prof. Wolfgang Woess

Fellow: Dr. Sara Brofferio (1.3.2003 – 31.8.2004, now Univ. Paris - Orsay)

External collaborator: Dr. Sébastien Blachère (Université de Marseille - I)

D. Random Walks, Random Configurations, and Horocyclic Products

FWF (Austrian Science Fund) Projekt Nr. P19115-N18

1.10.2006 – 31.3.2011

Project coordinator: Prof. Wolfgang Woess

Project personnel: Dipl.-Ing. Wilfried Huss (PhD student, 1.10.2006 – 15.9.2008 and 1.8.2009 – 30.9.2009, now Univ. Siegen)
Dr. Sebastian Müller (1.5.2007 – 30.9.2007, now Univ. Marseille)
Dr. James Parkinson (1.11.2007 – 15.12.2008, now Sydney Univ.)
Dr. Agelos Georgakopoulos (15.4.2009 – 31.3.2011, then Univ. Ottawa)
Mathias Hamann (PhD student, 16.9.2010 – 15.12.2010, now Univ. Hamburg)
Konrad Kolesko (PhD student, 16.9.2010 – 15.12.2010, now Univ. Wroclaw)

External collaborators: Dr. Sara Brofferio (Université de Paris-Orsay),
Prof. Laurent Bartholdi (EPFL Lausanne),
Prof. Donald Cartwright (University of Sydney)
Prof. Vadim A. Kaimanovich (International University Bremen),
Prof. Laurent Saloff-Coste (Cornell University).

F. Branching Random Walks in Probability, Analysis, Algebra, and Geometry

DFG (Deutsche Forschungsgemeinschaft) grant

1.10.2007 – 30.9.2009

Grant recipient: Dr. Sebastian Müller (now Univ. Marseille)

Grant supervisor: Prof. Wolfgang Woess

G. Random Walks on Free Products, Amalgamated Products, Trees and Strings over an Infinite Alphabet

DFG (Deutsche Forschungsgemeinschaft) grant

1.9.2009 – 1.3.2010

Grant recipient: Dr. Lorenz Gilch (now TU Graz)
Grant supervisor: Prof. Wolfgang Woess

H. Asymptotic Behaviour of Random Walks and Branching Random Walks on Free Products

Doc fForte PhD grant of the Austrian Academy of Sciences

1.4.2010 – 1.4.2012

Grant recipient: Elisabetta Candellero
Grant supervisor: Prof. Wolfgang Woess

I. Doctoral Program "Discrete Mathematics"

FWF (Austrian Science Fund) Doktoratskolleg-plus Nr. W1230-N13

1.5.2010 – 30.4.2014

DK speaker: Prof. Wolfgang Woess
10 faculty members from TU Graz and the Universities of Graz and Leoben, 10 FWF-funded PhD students, up to 11 associated PhD students. For details, see §3 above and
`http://www.math.tugraz.at/discrete`

6) Refereeing

Refereeing of
NSF research grant applications (1989,1990, 1992),
Cofin grant applications (Italy, comparable with NSF grants, 2001),
EU Marie Curie fellowship application ,
Israel Science Foundation research proposal (2005),
DFG (Deutsche Forschungsgemeinschaft) project application (Germany, 2006). ÖAD (Österreichischer Akademischer Austauschdienst) project applications (2010, 2011).

Evaluation of the Mathematical Institute of the Slovak Academy of Sciences (2007)

Assessment regarding various promotions to professor positions, resp. tenured positions at various US universities.

External committees:
Committee for prof. position, Univ. Neuchatel, Switzerland.
PhD committees at Univ. Brest, Univ. Toulouse, Univ. Paris 6,
Scuola Normale Superiore Pisa, Univ. Rome-I.

Throughout the years: Referee of papers presented for publication in a great variety of mathematics journals. A (long) list can be found on the personal webpage
`http://www.math.tugraz.at/~woess`

Reviewer for Mathematical Reviews and Internationale Mathematische Nachrichten.

7) Publications since 2006

Books

[1] W. Woess: *Random Walks on Infinite Graphs and Groups,* Cambridge Tracts in Mathematics **138**, Cambridge University Press, 334+xi pages, 2000. Paperback re-edition 2008.

[2] W. Woess: *Denumerable Markov Chains - Generating Functions, Boundary Theory, Random Walks on Trees,* EMS Textbooks in Mathematics, European Mathematical Society Publishing House, xviii+351 pages, 2009.

[3] D. Lenz, F. Sobieczky, and W. Woess (editors): *Random Walks, Boundaries, and Spectra.* Proceedings of two workshops, Graz and St. Kathrein 2009. Progress in Probability, Birkhäuser, in press.

Research papers

[4] L. Saloff-Coste and W. Woess: *Transition operators on co-compact G-spaces,* Revista Matematica Iberoamericana **22** (2006) 747–799.

[5] S. Brofferio and W. Woess: *Positive harmonic functions for semi-isotropic random walks on trees, lamplighter groups, and DL-graphs,* Potential Analysis**24** (2006) 245–265.

[6] R. Ortner and W. Woess: *Non-backtracking random walks and cogrowth of graphs,* Canadian J. Math. **59** (2007) 828–844.

[7] A. Karlsson and W. Woess: *The Poisson boundary of lamplighter random walks on trees,* Geometriae Dedicata **124** (2007) 95–107.

[8] D. I. Cartwright and W. Woess: *The spectrum of the averaging operator on a network (metric graph),* Illinois J. Math. **51** (2007) 805–830.

[9] L. Bartholdi, M. Neuhauser and W. Woess: *Horocyclic products of trees,* J. European Math. Society **10** (2008) 771–816.

[10] L. Saloff-Coste and W. Woess: *Computations of spectral radii on cocompact G-spaces,* Contemporary Math. **484** (2009) 195–218.

[11] F. Lehner, M. Neuhauser and W. Woess: *On the spectrum of lamplighter groups and percolation clusters,* Mathematische Annalen **342** (2008) 69–89.

[12] A. Bendikov, L. Saloff-Coste, M. Salvatori and W. Woess: *The heat semigroup and Brownian motion on strip complexes,* Advances in Mathematics **226** (2011) 992–1055.

[13] W. Huss, E. Sava and W. Woess: *Entropy sensitivity of languages defined by infinite automata, via Markov chains with forbidden transitions,* in print, Theoretical Computer Science.

[14] T. Ceccherini-Silberstein and W. Woess: *Context-free pairs of groups. I - Context-free pairs and graphs,* preprint.

[15] W. Woess: *Context-free pairs of groups. II - Cuts, tree sets, and random walks,* preprint.

[16] M. Peigné and W. Woess: *Stochastic dynamical systems with weak contractivity properties. With a chapter featuring results of Martin Benda,* preprint.

7) Ten publications considered most important, prior to 2006

(In order to avoid repetitions from the above list, this only contains publications prior to 2006!)

[A] W. Woess: *Nearest neighbour random walks on free products of discrete groups,* Bollettino

Unione Mat. Italiana **5-B** (1986) 691–982.

[B] M. A. Picardello and W. Woess: *Martin boundaries of random walks: ends of trees and groups,* Transactions Amer. Math. Soc. **302** (1987) 185–205.

[C] W. Woess: *Boundaries of random walks on graphs and groups with infinitely many ends,* Israel J. Math. **68** (1989) 271–301.

[D] P. M. Soardi and W. Woess: *Amenability, unimodularity, and the spectral radius of random walks on infinite graphs,* Math. Zeitschrift **205** (1990) 471–486.

[E] C. Thomassen and W. Woess: *Vertex-transitive graphs and accessibility,* J. Combinatorial Th., Ser. B. **58** (1993) 248–268.

[F] W. Woess: *Fixed sets and free subgroups of groups acting on metric spaces,* Math. Zeitschrift **214** (1993) 425–440.

[G] D. I. Cartwright, V. A. Kaimanovich and W. Woess: *Random walks on the affine group of local fields and of homogeneous trees,* Ann. Institut Fourier (Grenoble) **44** (1994) 1243–1288.

[H] W. Woess: *Random Walks on Infinite Graphs and Groups,* Cambridge Tracts in Mathematics **138**, Cambridge University Press, 334+xi pages, 2000.

[I] V. A. Kaimanovich and W. Woess: *Boundary and entropy of space homogeneous Markov chains,* Ann. Probab. **30** (2002) 323–363.

[J] S. Brofferio and W. Woess: *Green kernel estimates and the full Martin boundary for random walks on lamplighter groups and Diestel-Leader graphs,* Annales Inst. H. Poincaré (Prob. & Stat.) **41** (2005) 1101–1123.

14. Curriculum vitae and publications of Agelos Georgakopoulos

1) Education and employment

Born in Athens (Greece) 26 April 1979.

Studies of Electrical and Computer Engineering at the National Technical University of Athens.

2003: Diploma (≡ Master) degree.

2003–2006: Ph. D. studies at the University of Hamburg (supervisor: Reinhard Diestel).

2006 PhD degree in Mathematics, with disctintion.

2006–2009 Postdoc with Prof. R. Diestel at the University of Hamburg within the GIF projekt "Duality in Infinite Combinatorics".

2009–2011 Postdoc with Prof. W. Woess at TU Graz within the FWF projekt "Random Walks, Random Configurations, and Horocyclic Products".

2) Distinctions

1997: Third prize at the greek national mathematical competition for pupils

2005 – 2006: Merit scholarship ("Leistungsstipendium für ausländische Studierende") of the University of Hamburg

May 2008: Honorary mention within the framework of the Richard Rado prize (The only honorary mention).

3) Publications of A. Georgakopoulos

[1] Georgakopoulos, A.: *Infinite highly connected planar graphs of large girth,* Abh. Math. Sem. Univ. Hamburg **76** (2006) 235–245.

[2] Georgakopoulos, A.: *Connected but not path-connected subspaces of infinite graphs,* Combinatorica **27** (2007) 683–698.

[3] Aharoni, R., Georgakopoulos, A., Berger, E., and Sprüssel, Ph.: *Strongly maximal matchings in infinite weighted graphs,* Electron. J. Combin. **15** (2008) Research Paper 136, 18 pp.

[4] Aharoni, R., Georgakopoulos, A., and Sprüssel, Ph.: *Perfect matchings in r-partite r-graphs,* European J. Combin. **30** (2009) 39–42.

[5] Georgakopoulos, A.: *Infinite Hamilton cycles in squares of locally finite graphs,* Advances in Mathematics **220** (2009), no. 3, 670–705.

[6] Georgakopoulos, A.: *Topological circles and Euler tours in locally finite graphs,* Electron. J. Combin. **16** (2009) Research Paper 40, 16 pp.

[7] Georgakopoulos, A.: *A short proof of Fleischner's theorem,* Discrete Math. **309** (2009) 6632–6634.

[8] Georgakopoulos, A., and Sprüssel, Ph.: *Geodetic topological cycles in locally finite graphs,* Electron. J. Combin. **16** (2009) Research Paper 144, 18 pp.

[9] Bruhn, H., Diestel, R., Georgakopoulos, A., and Sprüssel, Ph.: *Every rayless graph has an unfriendly partition.* In *European Conference on Combinatorics, Graph Theory and Applications (EuroComb 2009),* 279–281, Electron. Notes Discrete Math. **34**, Elsevier, Amsterdam, 2009.

[10] Georgakopoulos, A.: *Graph topologies induced by edge lengths,* preprint (2009).

[11] Georgakopoulos, A.: *Uniqueness of electrical currents in a network of finite total resistance,* J. Lond. Math. Soc. **82** (2010) 256–272.

[12] Georgakopoulos, A.: *Lamplighter graphs do not admit harmonic functions of finite energy,* Proc. Amer. Math. Soc. **138** (2010) 3057–3061.

[13] DeVos, M., Georgakopoulos, A., Mohar, B., and Šámal, R.: *An Eberhard-like theorem for pentagons and heptagons,* Discrete Comput. Geom. **44** (2010) 931–945.

[14] Aharoni, R., Berger, E., Georgakopoulos, A., Perlstein, A., and Sprüssel, Ph.: *The max-flow min-cut theorem for countable networks,* J. Combin. Theory Ser. B **101** (2011)1–17.

[15] Georgakopoulos, A.: *Cycle decompositions: from graphs to continua,* preprint (2010).

[16] Georgakopoulos, A.: *A group has a flat Cayley complex if and only if it has a VAP-free Cayley graph,* preprint (2010).

[17] Georgakopoulos, A.: *The planar cubic Cayley graphs of connectivity 2,* preprint (2010).

[18] Georgakopoulos, A.: *The planar cubic Cayley graphs,* preprint (2011).