Random Walks and Geometry (ESI, June 18 – July 13, 2001)

ABSTRACTS OF TALKS

Martin BARLOW

Which values of the volume growth and anomalous diffusion exponents are possible?

The behaviour of a random walk on a suitably regular graph (or fractal) is captured by two exponents: the volume growth d_f and the space-time scaling d_w. For the Euclidean lattice $\mathbb{Z}^d$ one has $d_f = d$, $d_w = 2$. The case when $d_w > 2$, which arises for fractal lattices, is often called "anomalous diffusion". It is well known (and quite easy to prove) that in general $1 \leq d_f$, $2 \leq d_w \leq 1 + d_f$. In this talk we will show that these are the only contraints, by constructing for each (α, β) satisfying the inequalities above a graph (which is regular enough to satisfy an elliptic Harnack inequality) with $d_f = \alpha$, $d_w = \beta$.

Udo BAUMGARTNER

Finding a domicile for any totally disconnected locally compact group

For every totally disconnected locally compact group G we construct a topological space acted upon by the group of bicontinuous automorphisms of G. The construction parallels those of the spherical building of a semisimple group over a local field. Basic to the construction are "root groups" and analogues of split tori.

Sebastien BLACHERE

Cut times for random walk on discrete groups having polynomial growth

The study of cut times for symmetric random walks on finitely generated groups, with bounded increment, was almost fulfilled by Lawler, James and Peres. The last unknown case is when the group has polynomial growth of degree 3 or 4. In that case, we prove prove the existence of infinitely many cut times, almost surely, using a specific study of the Heisenberg group.

Sara BROFFERIO

How a centered random walk on the affine group goes to infinity

We consider the processes obtained by (left and right) products of random i.i.d. affine transformations of the Euclidean space $\mathbb{R}^d$. Our main goal is to describe the geometrical behavior at infinity of the trajectories of these processes in the most critical when the dilatation of the random affinities is centered. Then we derive a proof of the uniqueness of the invariant Radon measure for the Markov chain induced on $\mathbb{R}^d$ by the left random walk and prove a stronger property of divergence for the process induced by the right random walk.

Alexander BUFETOV

Markov operators and pointwise convergence of spherical averages for actions of free groups

A measure-preserving action of a free group can be viewed as a stationary Markov process. This approach allows one to obtain various statements on convergence of spherical averages for actions of free groups, and, in particular, gives a new proof of the Nevo – Stein theorem.

Angeles CARMONA

Boundary value problems on networks: some application to random walks

In this communication we consider random walks on networks and study some concepts related to their as the mean exit time and the escape probability, obtaining explicit expressions for that concepts. The mean exit time from a subset F of a network is the solution of certain Dirichlet problem. That solution can been characterized as the equilibrium measures for the subset F. On the other hand, the escape probability can be obtained either as the energy of the solution of a Dirichlet problem or from the solution of a Poisson equation. So, we analyze general boundary value problems on networks. For that, we define the involved discrete operators such as the gradient, the divergence and the laplacian. After studying the existence and uniqueness of such a problems, and in order to obtain closed expressions and qualitative properties of their solutions, we construct explicit expressions of its Green functions in terms of equilibrium measures for the involved subsets. The equilibrium measures can be obtained by applying the extremal charge method developed by the authors in previous works.

Donald CARTWRIGHT

Isotropic random walks on $\tilde{A}_n$ buildings
(joint work with Wolfgang Woess)

In a paper published in 1978, Stanley Sawyer studied random walks on homogeneous trees which were "isotropic", meaning that the transition probabilities $p(x, y)$ depend only on the graph distance between the vertices x and y. This definition generalizes naturally to $\tilde{A}_n$ buildings (locally finite and thick); now $p(x, y)$ depends on n "distances". Most of Sawyer's results generalize to this context. The simple random walk on the vertices of such a building is isotropic. Random walks on $SL(n+1, F)$, F a local field, which are bi-$SL(n+1, O)$-invariant lead to isotropic random walks of buildings.

Anna ERSCHLER (DYUBINA)

Random walks on amenable groups and harmonic functions on the universal cover of a Riemannian manifold

We construct examples of random walks with positive entropy on finitely presented amenable groups. As a corollary we get that there exists a compact Riemannian manifold with amenable fundamental group such that its universal cover is not Liouville. We also construct examples of simple random walks on groups and of Brownian motion on universal covers of compact Riemannian manifolds with intermediate drift growth.

Steven N. EVANS

Pinching and twisting Markov processes

A number of Markov processes on "exotic" state-spaces arise from partially collapsing an original Markov process by a pinching operation that identifies points of the original state space via an equivalence relation. The Markovianity of the new process is ensured by a suitable randomising twist over the uncollapsed region of the state space each time the process leaves the collapsed region. Examples of this construction are Walsh's spider (and diffusions on more general graphs), diffusions on objects such as buildings, and certain diffusions that arise in stochastic averaging of PDE. This talk reports on some recent work with Richard Sowers on the construction and study of such Markov processes.

Tim R. FIELD

Stochastic Hamilton-Jacobi theory on manifolds - the emergence of wave-functions

A principle of least action for a non-degenerate diffusion process on an arbitrary manifold is presented, in terms of a final value stochastic optimal control problem, such that the kinetic Lagrangian is quadratic in the Ito forward drift of the process. The dynamics governing the action emerges as a stochastic Hamilton-Jacobi condition and is expressed in terms of the geometry determined by the Levi-Civita connection of the diffusion tensor. The stochastic dynamics, non-linear in the action, is recast in linear form in terms of a pair of real valued wave-functions that are exponential in the action.

David FISHER

Local rigidity of group actions on homogeneous manifolds

(joint work with G.A. Margulis)

Let G be a semisimple real Lie group, with no compact factors and all simple factors of real rank at least two. We prove that many actions of G, and also lattices in G, on compact homogeneous manifolds are locally rigid. These results represent a signifigant advance over previously known results in that no assumption on hyperbolicity of the action is needed. For example, the results apply to the action of $SL(n, Z)$ on $T^n \times T^m$ where the action on the first factor is standard and the action on the second factor is trivial.

S. M. FROLOVITCHEV

Asymptotic solutions of the boundary value problems for Markovian processes with a small parameter

We construct an asymptotic solution of the an asymptotic solution of the boundary value problem for a diffusion process with a small parameter on the half-line. Also we construct an asymptotic solution of the boundary value problem for a jump process with small frequent jumps on the half-line.

Pawel GORA

Absolutely continuous invariant measures for random maps with position dependent probabilities

A random map is discrete-time dynamical system in which one of a number of transformations is randomly selected and applied at each iteration of the process. Usually the map τ_k is chosen from a finite collection of maps with constant probability p_k. In this talk we allow the probabilities p_k's to be functions of position. In this case the random map cannot be considered to be a skew product. The main result provides a sufficient condition for the existence of an absolutely continuous invariant measure for position dependent random maps on [0,1]. Geometrical and topological properties of sets of absolutely continuous invariant measures, attainable by means of position dependent random maps, are studied theoretically and numerically.

Ursula HAMENSTAEDT

Ergodic properties of Gibbs measures on nilpotent covers

Let M be a closed negatively curved manifold with fundamental group F. Every Gibbs equilibrium state of the geodesic flow on M defines a measure class on the ideal boundary of the universal covering of M. We use random walks to show that this measure class is ergodic under the action of every normal subgroup N of F such that F/N is nilpotent.

Irene HUETER

Mean square displacement exponent of the self-avoiding walk in all dimensions

We will consider (A) the point process of self-intersections defined via certain paths of length n of the symmetric simple random walk in the d-dimensional integer lattice and (B) a weakly self-avoiding cone process relative to the point process. The asymptotic expected distance of the process in (B) can be calculated rather explicitly as n tends large, at least if the point process is 1/2-shaped, in which case it can be shown to equal the expected distance of the weakly self-avoiding walk in the lattice from the starting point. From these results, the mean square displacement exponent of the self-avoiding walk in the d-dimensional lattice is immediately collectable.

Wojciech JAWORSKI

Boundaries of random walks and strongly approximately transitive actions of locally compact groups

The talk will discuss an approach to the theory of boundaries of random walks on locally compact groups based on the concept of a strongly approximately transitive group action.

Anders KARLSSON

Multiplicative ergodic theory and Busemann functions

We consider certain random products (or ergodic cocycles) of isometries (or 1-Lipschitz maps) of a metric space and formulate an a.e. convergence statement in terms of Busemann functions. In a joint work with Margulis we proved that such a statement holds true in the case where the metric space is nonpositively curved. This result generalizes Oseledec's multiplicative ergodic theorem (which in turn generalizes Birkhoff's ergodic theorem) and a random ergodic theorem of Beck-Schwartz (which in turn generalizes von Neumann's ergodic theorem). It also leads to an identification of the Poisson boundary of an arbitrary discrete group of isometries of a Cartan-Hadamard manifold (with a reasonable measure on the group).

Inkang KIM

Affine action and Margulis invariant

We show that Margulis invariant determines the affine deformation in $SO(n+1,n)\times \mathbb{R}^{2n+1}$ up to translational conjugacy. Also we show that an affine deformation of a lattice in $SO(2,1)$ in $\mathbb{R}^4$ with a reducible representation is not proper under the suitable condition. This is an analogue of quasifuchsian deformation of Fuchsian groups.

Anatoly N. KOCHUBEI

Stochastic processes over non-Archimedean fields

The field of p-adic numbers and related structures (its ring of integers, group of units, finite and infinite extensions etc.) constitute a class of algebraic structures admitting rich harmonic analysis, which can be employed in the construction and study of stochastic objects. The talk will include a brief review of various methods of constructing Markov processes over p-adics, properties of p-adic stable processes, stochastic differential equations, and a characterization of stable p-adic random variables in terms of a limit theorem for sums of independent identically distributed p-adic random variables. A more detailed account will be given of a non-Archimedean version of infinite-dimensional analysis, in which we consider measures and stochastic processes on a space conjugate to an infinite extension of the field of p-adic numbers.

Adam KORANYI

Is there an H^p theory on buildings?

The H^p theory of harmonic functions on the unit disc has been generalized in a fairly starightforward way to Riemannian symmetric spaces and buildings of rank one. The problem of extending to higher rank has been open for a long time. There are many indications and partial results that such an extension is possible, but some essential elements are missing. Prominent among these is an appropriate generalization of the Luzin area integral. The present state of affairs will be discussed, together with some conjectures.

Bernhard KROEN

Decomposition and analytic continuation of Green functions and asymptotic transition probabilities on self-similar graphs

Grabner and Woess used a substitution method and singularity analysis of probability generating functions on the Sierpiński graph to calculate the n-step return probabilities to the origin asymptotically. We introduce an infinite class of self-similar graphs for which we can apply these techniques and obtain the same results.

Furthermore we discuss an algorithm which yields the analytic continuations of any Green on these graphs. An application of this algorithm is the explicit computation of the spectrum of self-similar graphs.

Gregory F. LAWLER

Conformal invariance and continuum limits of two-dimensional systems

I will survey work done in collaboration with Oded Schramm and Wendelin Werner in understanding the continuum limits of random walk paths, nonintersecting random walks, boundaries of percolation clusters, loop-erased random walks, and self-avoiding walk in two dimensions. This is work in progress and I will discuss both theorems and conjectures. The key tools are: conformal invariance, universality, and a process introduced by Schramm, the Stochastic Loewner Evolution.

Russell LYONS

Uniform spanning forests and the geometry of random walks and groups

A number of aspects of random walks on groups arise in a new perspective when random walks are used to create uniform spanning forests. We shall illustrate this with respect to intersections of random walks, existence of harmonic Dirichlet functions, Poisson boundary, amenability, conformal invariance and Brownian motion in two dimensions, and L^2-Betti numbers.

Gregory MARGULIS

Recurrence properties of random walks on locally symmetric spaces

This is a joint work with A.Eskin. We prove that, under certain conditions, random walks on finite volume locally symmetric spaces return to "large" compact sets with the probability "close to 1".

Fabio MARTINELLI

Asymmetric simple exclusion and interfaces of the quantum XXZ model

I will consider the spin 1/2 anisotropic Heisenberg model XXZ in a cylinder of $\mathbb{Z}^3$ with axis in the 111 dierction. The ground states can be computed explicitly and it turns out that the Hamiltonian is unitarily equivalent to the generator of an asymmetric simple exclusion. I will then analyze this last model in some details and prove in particulrily that its relaxation time scales diffusively in the linear size of the basis of the cylinder.

Pierre MATHIEU

Log-Sobolev and spectral gap inequalities for the knapsack problem

We study a random walk moving on a discrete cube of large dimension and subject to a linear constraint. Under the assumption that more than the half of the points in the cube satisfy the constraint, we prove log-Sobolev and Poincare inequalities with constants of the correct order.

Stanislav MOLCHANOV

Random walks on some finite and compact groups and testing of RNG's

Let $X_1, X_2, \dots, X_n, \dots$ be i.i.d.r.v. (for instance, symmetric Bernoulli or uniform on $\mathbf{S}^1 = [0,1)$). Such sequences are typical outputs of the algorithmical (pseudorandom) or the physical random generators (RNG's). Successive words of the fixed length k, $W_1 = [X_1 \dots X_k]$, $W_2 = [X_2 \dots X_{k+1}]$, $\dots$, $W_n = [X_n \dots X_{n+k}]$, $\dots$ form, of course, a Markov chain on very simple groups like $\mathbb{Z}_2^k$ (Bernoulli case) or $\left(\mathbf{S}^1\right)^k = T_k$ (uniform r.v.). Occupation measures for such random walks satisfy the LLN and the CLT. The central problem is to find the effective description of the spectra of the covariance operators of the gaussian fluctuations of the occupation measures. We'll discuss the solution of this problem. One of the applications of this result is the construction of the optimal functionals (in the spirit of the principal components idea in the factor analysis) for testing of RNG's. Results of the testing will be also presented.

Michail MONASTYRSKY

Monodromy representations of braid groups and classical and quantum two-dimensional models

We discuss some connections between special representations of Braid groups,topological invariants of related links and knots from the one hand and random walks on graphs from the other hand. Monodromy representation of braid groups related with coprrelation functions of 2-dim conformal field models. Some connections between these apparently different topics will be established.

Tatiana NAGNIBEDA

Ergodic properties of boundary actions

We study the action of a subgroup H of a free group F on the Poisson boundary of the simple random walk on F. We use methods of combinatorial group theory to identify the conservative and dissipative parts of the action, and give necessary and sufficient conditions of conservativity in terms of geometry of the quotient F/H.

Volodymyr NEKRASHEVYCH

Limit spaces of self-similar group actions

We will present a notion of a limit space of a contracting self-similar action of a group on a rooted regular tree. This is a connected compact finite-dimensional topological space often having a fractal nature. It is constructed as a quotient of the space of infinite sequences by a natural equivalence relation defined by the action of the group. It also can be interpreted as the boundary of a naturally defined hyperbolic space. Connections of the limit spaces with the random walks on the Schreier graphs of the group will be discussed.

Sam NORTHSHIELD

Cogrowth of arbitrary graphs

For a graph G, the "growth" of G is a constant c such that metric balls in the universal covering tree T grow like c^n. The "cogrowth" of G is a constant c_R such that the intersections of the preimage in T of a vertex of G with metric balls grows like c_R^n. For example, the growth of a d-regular graph is $d-1$ and its cogrowth is strictly less than $d-1$ if and only if G is non-amenable. I will discuss some recent progress toward an analogous statement for non-regular graphs. Some of this progress is based on properties of a family of Laplacians related to zeta functions of finite graphs.

Dimitri PETRITIS

Random walks on randomly oriented lattices

(joint work with Massimo Campanino)

Given a sequence of $\{-1, 1\}$-valued variables indexed by the ordinate of the lattice points on $\mathbb{Z}^2$, called the environment, we study simple random walks on the square lattice whose horizontal directions are one-way and their actual left- or right-going is determined by the environment variable. We establish results concerning recurrence-transience and speed of the simple random walk for various deterministic or i.i.d. environments.

Christophe PITTET

On an inequality of Varopoulos for finitely generated groups and the question of its optimality

A finitely generated group with exponential growth has a heat decay on the diagonal which goes in time at least as fast as an exponential of a cubic root. Alexopoulos showed that this is optimal in the case of polycyclic groups. We will present a generalization of this result, based on technics elaborated with Coulhon and Grigor'yan and some recent work with Saloff-Coste, which brings some light on the structure of the groups for which the above estimate is optimal.

David REVELLE

Rate of escape of random walks on groups

We find asymptotics for the limsup and liminf of the rate of escape for a variety of random walks on groups of exponential growth. In the examples that we study, these behave (with log log correction factors) like n^a, for a of the form $1 - 2^{-k}$. The bulk of our examples have already been considered by Anna Dioubina, who found similar behavior for the expected value of the rate of escape.

Ben-Zion RUBSHTEIN

On a class of one-sided Markov shifts

We consider one-sided Markov shifts which can be represented as finite extensions of the one-sided (p, q)-Bernoulli shift.

Richard SHARP

A local limit theorem for closed geodesics and homology

A compact negatively curved manifold has a countable infinity of closed geodesics. We shall discuss limit laws for the distribution of closed geodesics lying in a prescribed homology class. Subject to a pinching condition, we prove a local limit theorem, which is uniform as the homology class varies.

Nikita SIDOROV

Unique beta-representations of real numbers and dynamics

In this talk we would like to consider the set of numbers which admit a unique representation in a given non-integer radix base and the underlying dynamics of "maps with holes".

Stanislav SMIRNOV

Conformal invariance of critical percolation

We will discuss critical site percolation on triangular lattice in the plane. We will prove conformally invariant Cardy's formula (conformal field theory prediction for the probabilities of crossings) and discuss the construction of the (conformally invariant) continuum scaling limit.

Rita SOLOMYAK

Invariant measures for some equivalence relations

We discuss invariant measures for tail equivalence relation, or orbit equivalence relation for some skew-products. Joint work with J. Aaronson, H. Nakada and O. Sarig.

Jeff STEIF

Dynamical sensitivity of randomness

We consider a sequence of random bits, where each bit is refreshed independently according to a Poisson clock. At any fixed time t, the resulting sequence has the same law as at time 0, but there can be exceptional random times at which certain randomness criteria are not satisfied. We prove that there are no such exceptional times for the law of large numbers and the law of the iterated logarithm but that there are such times at which run lengths are exceptionally long. A multifractal analysis of exceptional times for both run lengths and for bit prediction is obtained. Finally, random walks are also studied. This is joint work with Itai Benjamini, Olle Haggstrom and Yuval Peres.

Domokos SZASZ

Recurrence of the planar Lorentz process by dynamical methods

(joint talk with Tamas Varju)

In $2 - D$, the diffusive limit of the Lorentz process with a periodic configuration of scatterers is the Brownian motion if the free path is bounded (the so-called finite horizon case). Once so, then — à la Pólya — the recurrence of the process is a natural property to expect. Indeed, ergodic theoretic proofs were obtained independently by J.-P. Conze (1999, EThDS) and Klaus Schmidt (1998, CR ASP). Here local limit theorems for the planar Lorentz process with a finite horizon are presented which also imply the recurrence of the process. We also expect that these methods based on the theory of dynamical systems extend to the infinite horizon case but so far this has not been done.

A. UGLANOV

On potential theory in infinite-dimensional spaces

We present some results on boundary value problems in a locally convex space. The fundamental and Green's measures are introduced, the volume and surface potentials are defined, the formulas, expressing the solutions of Dirichlet problem for elliptic and parabilic equations, are given. As it known, boundary value problems (both in finite and infinite-dimensional cases) for elleptic and parabolic equations are closely connected to diffusion stochastic processes. On other hand, as it clarified recently, boundary value problems playes central role in calculus of variations on infinite-dimensional spaces (and this calculus, in turn, is useful for control of stochastic processes). In fact, we have obtained some results in indicated domains. The surface integration theory in a locally convex space, that has been constructed recently, is a main instrument of the investigation, therefore indicated results was not and just could not be obtained before. All our considerations and formulas are dimensional-invariant, and in conformity to finite-dimensional case they turn into classical ones.

John A. VELLING

Escape rates, growth rates and Hausdorff dimension — behavior at infinity of hyperbolic manifolds

For quotients of hyperbolic $(n+1)$-space, there is a very precise relationship between several invariants corresponding to a point in the minimal Martin boundary: (1) the growth rate of the extremal positive harmonic function, (2) the Hausdorff dimension of the supporting ergodic measure at infinity on the n-sphere, (3) the decay rate of the Green's function in the direction of the given point, and (4) the escape rate of geodesic flow in the direction of the point. It is conjectured that these can be linked to the Hausdorff dimension of harmonic measure in the Martin boundary. This conjecture can be established in special cases.

Raphael VOITURIEZ

Random walks on the braid group B_3 and magnetic translations in hyperbolic geometry

We study random walks on the three-strand braid group B_3, and in particular compute the drift, or average topological complexity of a random braid, as well as the probability of trivial entanglement. These results involve the study of magnetic random walks on hyperbolic graphs (hyperbolic version of the Harper-Hofstadter problem), what enables to build a representation of B_3 as generalized magnetic translation operators for the problem of a quantum particle on the hyperbolic plane.

George WILLIS

Totally disconnected locally compact groups

Work on a conjecture of Hofmann and Mukherjea concerning the concentration functions of a random walk has led to a structure theory for general totally disconnected locally compact groups. Recent developments will be described in the talk. Automorphisms of these groups have a canonical form analogous to the Jordan canonical form of a linear transformation and there are structures in the group which are analogues of eigenspaces and eigenvalues.

Anton ZORICH

Geometry and dynamics of flat surfaces

In numerous problems of dynamics (like billiards in rational polygons, topological dynamics of measured foliations on surfaces, interval exchange transformations, etc) it is natural to consider a flat metric on a Riemann surface, where all curvature of the surface is collapsed to several cone-type singularities. We consider a class of such flat metrics having trivial holonomy: parallel transport of a vector along a smooth closed path leaves the vector invariant. A surface endowed with such special flat metric is called a translation surface. I want to present several recent results of A.Eskin, M.Kontsevich, H.Masur and myself concerning geometry and dynamics on translation surfaces. In particular I shall discuss the amazing behavior of configurations of closed geodesics on translation surfaces, the quantitative aspects of the asymptotics of the number of closed geodesics of bounded length, and applications of these results to the billiards in "rectangular polygones".

Andrzej ZUK

Random walks and the Atiyah conjecture

We present a computation of the spectrum of the lamplighter group obtained using theory of finite automata. We explain how this computation yields a counterexample to a strong version of the Atiyah conjectures about the range of L^2-Betti numbers of closed manifolds.