GAG workshop
TUGraz 11-12 February 2019

C. Müllner
Title. Normal subsequences of automatic sequences.
Abstract. In 2013 Drmota, Mauduit and Rivat observed that the subsequence along the squares $(t(n^2))_{n\geq 0}$ of the Thue-Morse sequence $(t(n))_{n\geq 0}$ (that can be defined by $t(n) = s_2(n) \bmod 2$, where $s_2(n)$ denotes the binary sum-of-digits function) is a normal sequence on the alphabet $\{0, 1\}$. This means that every block $w \in \{0, 1\}$ appears with density 2^{-k}. The purpose of this talk is to discuss this result also from a more general point of view. The Thue-Morse sequence is a special case of an $k-$automatic sequence $(a(n))_{n\geq 0}$, that is, a sequence where the $n-$th element is the output of a deterministic finite state automaton, where the input is the base k expansion of n. Automatic sequence have a sub-linear subword complexity. This means that at most $c \cdot k$ different blocks of length k appear so they are far from being normal. Further- more, linear subsequences of automatic sequences are again automatic sequence and, therefore, have again sub-linear subword complexity. However, when we consider a subsequence $a(\phi(n))$, where $\phi(n)/n \to 1$ the situation can change completely - as $(t(n^2))_{n\geq 0}$ shows. Thus, we discuss the question for which subsequences - and for which automatic sequences - we may expect a normal sequence. Recent results in this direction, e.g., by Lukas Spiegelhofer and the author (who considered $\phi(n) = \lceil n^c \rceil$ for the Thue-Morse sequence, where $1 < c < 3/2$) and by the author (who considered block-additive functions like the Rudin-Shapiro sequence and $\phi(n) = n^2$) indicate that there might be a more general principle behind. Finally, we also consider the subsequences along primes of automatic sequences. It seems that showing normality for such sequences is out of reach. However, it is possible to describe the frequences of letters for these subsequences.

I. Bondarenko
Title. The duality of the affine actions on trees.
Abstract. Every action on a tree given by a (finite) automaton has an associated dual action given by the dual automaton. In this talk I will consider the affine groups of subrings of a global function field, construct their actions on a regular tree, and describe the dual action. In particular, this gives a natural family of bireversible automata and square complexes with interesting properties coming from the affine groups of global function fields.
The talk is based on a joint work with Dmytro Savchuk.

A. Perez
Title. Spectral properties related to spinal groups.
Abstract. Spinal groups were defined by L. Bartholdi, R. Grigorchuk and Z. Å uniÄ as a generalization of Grigorchuk's group, which features several in-

teresting properties like having intermediate growth or being amenable but not elementary amenable. In this talk we will first introduce this family of automata groups and describe the Schreier graphs associated with their action on the d-regular rooted tree. Later, we will consider the Markov operator on these graphs and, by exploiting the self-similarity given by the automata, we will find some of its spectral properties, including its spectrum explicitly. The talk will be based on an ongoing joint work with R. Grigorchuk and T. Nagnibeda.

T. Nagnibeda
Title. TBA.
Abstract.

E. Rodaro
Title. On some recent structural and algorithmic results for Automaton (Semi)groups.
Abstract. In this talk, I am going to survey some recent results regarding the structure and some algorithmic problems for automaton semigroups and groups. In particular, from the structural point of view we will concentrate on the finiteness and freeness problem and the interaction of these problems with the dynamics of these (semi)groups on the boundary Σ^{ω}.
This is a series of joint works with D. D'Angeli, D. Francoeur and J-F. Wächter.

J. P. Wächter
Title. Inverse and Partial Automaton Semigroups.
Abstract. Automaton groups are generated by invertible and complete automata. Usually, automaton semigroups are obtained by dropping the requirement that the automata must be invertible. However, we can also drop the requirement that the automata must be complete (but retain invertibility). This way, we obtain a natural presentation for a class of inverse automaton semigroups and, by also dropping the completeness requirement, another way to present automaton semigroups.

In the talk, we motivate the study of inverse automaton semigroups and semigroups generated by partial automata from a decision problem perspective and, afterwards, address two questions of a more structural nature. First: does the class of semigroups generated by partial automata coincide with the class of semigroups generated by complete automata? And second: is every inverse (partial) automaton semigroup generated by a partial, invertible automaton?

D. Francoeur
Title. On free subsemigroups in automata semigroups.
Abstract. Automata groups and semigroups can have many interesting and

exotic properties. For example, one can find among them groups of intermediate growth and infinite finitely generated torsion groups. One may wonder how the structure of the underlying automaton can influence these properties. In this talk, we will investigate one such connection, namely the link between the reversibility of the automaton and the existence of a free non-abelian subsemigroup.
This is joint work with Ivan Mitrofanov.

P. Gillibert
Title. Undecidability in automaton groups.
Abstract. An self-similar group is a subgroup of the automorphism group of a tree of all words (over a finite alphabet), such that the induced action on sub-trees are also in the group. An automaton group is a self-similar group satisfying a stronger condition: There is a finite set of generators, such that the action of generators on sub-trees are also in the set of generators. Equivalently the group is generated by the actions induced by a Mealy automaton (the set of states correspond to the set of of generators). Using automaton groups Aleshin constructed a simple example of a Burnside group. Later Grigorchuk gave an example of an automaton groups which is Burnside, just-infinite, amenable, non-elementary amenable and of intermediate growths, solving Milnor's problem and Day's problem. The word problem is solvable in automaton groups (Eilenbergâs reduction algorithm). However there is an automaton group with undecidable conjugacy problem (Sunic and Ventura). Several other relatively simple problems are also undecidable. We prove that the order problem is undecidable. From any Turing Machine, with a one-direction infinite tape, we constructs an automaton group. It simulates the Turing Machine in the following sense: Given any entry word there is an (explicitly constructed) element of the group such that the Turing Machine stops on this entry if and only if the order of the element is finite (then of order a power of 2). In particular, considering an automaton group constructed from a universal Turing Machine, there is no algorithm which decides whether or not an element is of finite order.

T. Godin
Title. The activity of automaton semigroups.
Abstract. .We define a new strict and computable hierarchy for the family of automaton semigroups, which reflects the various asymptotic behaviors of the state-activity growth. This hierarchy extends that given by Sidki for automaton groups, and also gives new insights into the latter. Its exponential part coincides with a notion of entropy for some associated automata. We prove that the Order Problem is decidable when the state-activity is bounded. The Order Problem remains open for the next level of this hierarchy, that is, when the state activity is linear. Gillibert showed that it is undecidable in the whole family. We extend the aforementioned hierarchy

to a semi-norm making it more robust and prove that Order Problem is still decidable for the first level of this new hierarchy.
Joint work with Laurent Bartholdi, Ines Klimann and Matthieu Picantin

C. Lindorfer
Title. The language of self-avoiding walks .
Abstract. Let $X = (VX, EX)$ be an infinite, locally finite, connected graph without loops or multiple edges. We consider the edges to be oriented, and EX is equipped with an involution which inverts the orientation. Each oriented edge isÂ labelled by an element of a finite alphabet Σ. The labelling is assumed to be deterministic: edges with the same initial (resp. terminal) vertex have distinct labels. Furthermore it is assumed that the group of label-preserving automorphisms of X acts quasi-transitively. For any vertex x, consider the language of all words over Σ which can be read along self-avoiding walks starting at x. We characterize under which conditions on the graph structure this language is regular or context-free. This is the case if and only if the graph has more than one end, and the size of all ends is 1, or at most 2, respectively. In particular this applies to Cayley graphs of finitely generated groups, which are necessarily virtually free and as a consequence context-freeness is not a group invariant.

D. Kahrobaei
Title. Some applications of arithmetic and graph groups in cryptography.
Abstract. In this talk we will give various examples of exponentially distorted subgroups in linear groups, including some new example of subgroups of $SL_n(\mathbb{Z}[x])$ for $n \geq 3$, and show how they can be used to construct symmetric-key cryptographic platforms.
This is a joint work with K. Mallahi-Karai to appear in the Journal of Groups Complexity, Cryptology, De Gruyter.

E. Ventura
Title. The degree of commutativity/nilpotency of an infinite group.
Abstract. There is a classical result saying that, in a finite group, the probability that two elements commute is never between 5/8 and 1 (i.e., if it is bigger than 5/8 then the group is abelian). We make an adaptation of this notion for finitely generated infinite groups (w.r.t. a fixed finite set of generators) as the limit of such probabilities, when counted over successively growing balls in the group. This asymptotic notion is a lot more vague than in the finite setting, but we are still able to prove some interesting results: (1) with some hypothesis the limit exists and is independent from the set of generators; and (2) a Gromov-like result: “for any finitely generated residually finite group G of subexponential growth, the commuting degree of G is positive if and only if G is virtually abelian“. We then give some generalizations in two directions: changing to other distributions (i.e., other

directions to infinity) and other equations (degree of r-nilpotency).
This is joint work with Y. Antolin, A. Martino, M. Tointon, and M. Valiunas