Topology and dynamics of closed one forms

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10:30–10:50

Consider a closed manifold M and a closed one form ω on M whose zeros are all non-degenerate. Fix a Riemannian metric on M and consider the vector field $X := -\sharp\omega$ corresponding to $-\omega$ via the Riemannian metric. Thus, locally, the one form is exact, $\omega = df$, f has Morse type critical points, and X is the negative gradient of f. However, globally the dynamics of X may be very different from the dynamics of a gradient vector field, for instance, X might have periodic trajectories. Suppose x and y are two zeros of ω of index difference 1, and let γ be a homotopy class of paths from x to y. Assuming Smale transversality, ie. stable and unstable manifolds of X intersect transversally, Novikov observed that there are at most finitely many trajectories from x to y in each homotopy class γ. Counting these trajectories with signs we obtain incidence numbers $\mathbb{I}^{\gamma}_{x,y} \in \mathbb{Z}$. Novikov conjectured that these numbers grow at most exponentially, more precisely, there exists a constant $C \geq 0$ so that

$$\left|\sum_{-\omega(\gamma)=t} \mathbb{I}^{\gamma}_{x,y}\right| \leq e^{Ct},$$

for all $t \in \mathbb{R}$. The sums above are finite and taken over all homotopy classes γ with $-\omega(\gamma) = t$. We will discuss a recent approach to this conjecture.