The BGK model with external confining potential: Existence, long-time behaviour and time-periodic Maxwellian equilibria

TUE/E3.1
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The basic issue is a question going back to an observation of Boltzmann himself that for an isotropic harmonic confining potential on the whole space, there are periodic in time (non constant) local Maxwellian solutions for the nonlinear Boltzmann equation.
This raises the questions: What are the stability properties of the solutions? Do they have any nontrivial domain of attraction?
We investigate these problems for the Boltzmann and in particular for the Bhatnagar-Gross-Krook (BGK) equation of the kinetic gas theory:

$$\partial_t f + v \cdot \nabla_x f - x \cdot \nabla_v f = M[f] - f\,, \qquad f(t=0) = f_0 \tag{1}$$

with $(t,x,v) \in (0,+\infty) \times \mathbb{R}^N \times \mathbb{R}^N$, where the *local Maxwellian* $M[f]$ is defined in terms of the velocity moments of f.
By an entropy argument and a parallel study of the moments' evolution we identify the domain of attraction of all these states, that implies necessary (and, for the BGK, sufficient) conditions on the initial datum to get the convergence to a periodic equilibrium. For the BGK model we also study global-in-time existence and long-time behaviour in L^1 and in Lyapunov sense for more general external potentials.

[1] R. Bosi, M. J. Cáceres, *The BGK model with external confining potential: Existence, long-time behaviour and time-periodic Maxwellian equilibria*, to appear in J. Stat. Physics.
[2] C. Cercignani *The Boltzmann equation and its applications*, Springer, 1988.
[3] L. Desvillettes, *Convergence to equilibrium in large time for Boltzmann and B.G.K equations*, Arch. Rational Mech. Anal. 110, 1 (1990), 73-91.
[4] B. Perthame, *Global existence to the BGK model of Boltzmann equation*, J. Differential Equations, 82, (1989), 191-205.