Asymptotics of return probabilities of random walks on free products of lattices

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17:00–17:20

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Suppose we are given a free product of the form $\mathbb{Z}^{d_1} * \mathbb{Z}^{d_2}$ with d_1 and $d_2 \in \mathbb{N}$. We consider random walks on $\mathbb{Z}^{d_1} * \mathbb{Z}^{d_2}$ governed by probability measures $\mu := \alpha_1\mu_1 + \alpha_2\mu_2$, where μ_1, μ_2 are irreducible probability measures on $\mathbb{Z}^{d_1}, \mathbb{Z}^{d_2}$ and $\alpha_1, \alpha_2 > 0$ with $\alpha_1 + \alpha_2 = 1$. We investigate the asymptotic behaviour of the n-step return probabilities of *nearest neighbour random walks*, that is, the asymptotic behaviour of the probability of returning to the starting point after n steps.

Gerl (see [2]) conjectured that the n-step return probabilities of two symmetric measures on a group have the same asymptotic type $\rho^n n^{-\lambda}$ (ρ is the spectral radius), that is, λ is a group invariant. Cartwright (see [1]) came to the astonishing result that for simple random walks on $\mathbb{Z}^d * \mathbb{Z}^d$ with $d \geq 5$ there are at least two possible types of asymptotic behaviour, namely $n^{-3/2}$ and $n^{-d/2}$.

This was the starting point for the present investigation, if the range of different asymptotic behaviour can still be wider.

With the help of *Darboux's method*, we prove that in the more general setting of free products of the form $\mathbb{Z}^{d_1} * \mathbb{Z}^{d_2}$, only the following asymptotic behaviour can occur: $\rho^n n^{-3/2}, \rho^n n^{-d_1/2}, \rho^n n^{-d_2/2}$.

[1] D. CARTWRIGHT: On the asymptotic behaviour of convolution powers of probabilities on discrete groups. *Monatshefte für Mathematik* **107** (1989), 287–290.

[2] P. GERL: A local central limit theorem on some groups. In: *The first Pannonian Symposium on Mathematical Statistics, Lect. Notes Statistics 8* (Springer, P.Révész et al.eds.), 1981, pp. 73–82.

[3] W. WOESS: *Random Walks on Infinite Graphs and Groups*. Cambridge University Press 2000.