The Infinity of Reason. G. Cantor’s antinomies of infinity and I. Kant’s transcendental philosophy

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Two antinomies arise in set theory of Georg Cantor (1845-1918). He tries to solve them with the introduction of three theorems (A, B and C), but empirically he cannot solve them, because he gets into regressive structures of justification. Immanuel Kant’s question for infinity is more differentiated, which I want to show in a ’topography of infinity-places’ in his ’critique of pure reason’. My thesis of the paper is, that Cantor’s antinomies could solved in a transcendental way by criticizing the different spheres. Furthermore I want to demonstrate that the arising and solution of antinomies is not only important for mathematics and philosophy, but of particular importance for the groundwork of philosophy of mathematics.

Georg Cantor (1845-1918) is not only founder of set theory, but furthermore founder of ’metaphysics of the actual infinity’. In his set theory arise two antinomies:

1. Antinomy: The first antinomy emerges from the acceptance of totality of all ordinal numbers Ω as an ordinal number. The outcome of this is an antinomy between the acceptance of totality of all ordinal numbers and the acceptance that there is no ordinal number beyond this totality.

2. Antinomy: The second antinomy emerges from the searching of supreme generic terms (genus supremum) of a system S of the transfinite set theory. The application of the law of power set shows that it is not a supreme generic term, because it transcends it and stands in conflict with it.

Cantor tries to solve both antinomies with an introduction of three theorems A, B and C, which fails, because he gets into regressive structures of justification. Also the ZFC-axiomatisation of set theory fails, because it tries to solve the antinomies in an empirical way with empirical methods and so gets into regressive structures of justification. Therefore I want to ask, if the antinomies are solvable in a transcendental way?

In contrast to Cantor Kant develops in his ’critique of pure reason’ a ’topography of infinity-places’. Especially in the first antinomy in his critique Kant asks for the finity and infinity of the world in time and space. Kant shows that there is no concept of infinity, only the notion of a rule of a regressus in indefinitum. Thus there is no concept of the totality of all ordinal numbers Ω or of the system S of all cardinal numbers. Therefore you must distinguish critically between totality and its individuals. Although there is no answer for the question of the existence of a totality of sets, you can belief in infinity.

Indeed Kant and Cantor opened the garden of infinity, where their infinities are only the smallest of bigger infinities. Therefore the critiques of set theory and transcendental philosophy are the same as the critiques of philosophy of mathematics. Concluding I want to ask, what it means for the groundwork of philosophy of mathematics.