Sums of large sets

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A classical result of Steinhaus asserts that if $A, B \subseteq \mathbb{R}$ are sets of reals with positive Lebesgue measure, then the "sumset" $A + B = \{a + b : a \in A, b \in B\}$ contains a non-empty open set. Renling Jin used non-standard analysis to derive an analogous statement which applies to the integers: If $A, B \subseteq \mathbb{Z}$ are sets of positive density, then $A + B$ is large in a certain topological sense, namely piecewise syndetic. Jin's Theorem has been generalized and refined in various directions by several authors (including Bergelson, Fish, Furstenberg, Keisler and Weiss). We will discuss different approaches which have proven to be fruitful. Indeed the methods applied in this context range from classical combinatorics over ultrafilters to techniques with an ergodic theoretic flavor.