How to count polyominoes and polycubes

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We have analyzed the growth in the number of polyominoes on a *twisted cylinder* as the number n of cells increases, by using computers [1]. The polyominoes on these surfaces are related to the classical polyominoes (connected subsets of a square grid) that lie in the plane. We have thus obtained the current best lower bound of 3.980137 on *Klarner's constant*, the growth rate of the number of polyominoes.

We have also obtained explicit formulas for the numbers *polycubes*, high-dimensional analogs of polyominoes, for those cases where the number d of dimensions that are spanned by a polycube is not much smaller than n. In particular, for $d = n-1$ (the largest possible value of d), there are $2^{n-1}n^{n-3}$ polycubes, for $d = n-2$, there are $2^{n-3}n^{n-5}(n-2)(2n^2-6n+9)$ polycubes, and for $d = n-3$, there are $2^{n-6}n^{n-7}(n-3)(12n^5-104n^4+360n^3-679n^2+1122n-1560)/3$. These formulas are based on a correspondence with directed spanning trees and on an inclusion-exclusion principle, counting certain "substructures" that may appear in polcubes [2]. Such formulas have been proposed without rigorous proofs in the statistical-physics literature [3].

[1] G. BAREQUET, M. MOFFIE, A. RIBÓ, G. ROTE: Counting polyominoes on twisted cylinders. *INTEGERS: The Electronic Journal of Combinatorial Number Theory* **6** (2006), article #A22, 37 pages.

[2] R. BAREQUET, G. BAREQUET, AND G. ROTE: Formulae and growth rates of high-dimensional polycubes. *Combinatorica*, to appear.

[3] P.J. PEARD AND D.S. GAUNT, $1/d$-expansions for the free energy of lattice animal models of a self-interacting branched polymer, *J. Phys. A: Math. Gen.* **28** (1995), 6109–6124.