Generalization of Newman's phenomenon for weighted sums of digits

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Let $n = n_0 + n_1 \cdot 2 + \cdots + n_r \cdot 2^r$ be the base 2 representation of n and $\gamma := (\gamma_0, \gamma_1, \gamma_2, \ldots)$ with $\gamma_i \in \{0,1\}$. We define the *weighted sum of digits* of n by

$$s_{\gamma,2}(n) := n_0 \cdot \gamma_0 + n_1 \cdot \gamma_1 + \ldots n_r \cdot \gamma_r \quad \text{and} \quad S(N,k) := \sum_{n=0, n\equiv k(3)}^{N-1} (-1)^{s_{\gamma,2}(n)}.$$

The investigation of such sums is motivated by the study of distribution properties of Niederreiter-Halton sequences.

For the unweighted sum of digits, i.e. $\gamma_j = 1$ for all j, a well-known result of Newman [1] says that

$$\exists c > 0 \text{ such that for all } N \in \mathbb{N}: \qquad S(N,0) > c \cdot N^{\frac{\log(3)}{\log(4)}}. \tag{1}$$

Drmota and Stoll [2] have shown that in the unweighted case

$$S(N,1) < 0 \text{ for } N \geq 2 \text{ and } S(N,3) \leq 0 \text{ for } N \geq 3. \tag{2}$$

In this talk we study analogous results for weighted sums of digits. We show that Newman's result does not hold if $\gamma_j = 0$ for at least one j. However for certain weighted sums of digits we can generalize the results (1) and (2) in the sense that

$$\exists c' > 0, N_0 \in \mathbb{N} \text{ such that for all } N > N_0: \qquad |S(N,k)| > c' \cdot N^{\frac{\log(3)}{\log(4)}}. \tag{3}$$

We will discuss under which conditions on the weights γ result (3) is obtained.

[1] D. J. NEWMAN: *On the number of binary digits in a multiple of three*. Proc. Amer. Math. Soc. 21, 1969, pp. 719-721.

[2] M. DRMOTA AND TH. STOLL: *Newman's phenomenon for generalized Thue-Morse sequences*. Discrete Math. 308, 2008, pp. 1191-1208.