A connected fractile of infinite connectivity

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In the complex plane $\mathbb{C}$, let A_0 be the unit interval with endpoints 0 and 1, and let $A_{(1)}$ be the polygonal path with nodes 0, $\frac{1+i\sqrt{3}}{4}$, $\frac{1}{2}$, $\frac{3-i\sqrt{3}}{4}$, 1. Let f be the operator which, applied to a segment $E_{(0)}$ in $\mathbb{C}$, replaces it by a similar copy $E_{(1)} = f(E_{(0)})$ with the same endpoints. Denote by $f^{(n)}$ the n-th iterate of f. The limit set (with respect to the Hausdorff metric) $A_{(\infty)} = \lim_{n\to\infty} f^{(n)}(A_{(0)}$ is a space-filling curve which is the closure of its interior and the union of four half-size copies of itself, intersecting only in their boundaries. Although $A_{(\infty)}$ is of infinite connectivity, copies of it tessellate the plane. It is related to the set of Eisenstein fractions and has a boundary of Hausdorff dimension $\frac{\log 3}{\log 2}$.