Energy minimization in thin-film micromagnetics

THU/P2
16:30–16:50

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The steady state of a magnetization $\mathbf{M}$ of a ferromagnetic sample Ω was first described by Landau and Lifschitz as the solution of a certain minimization problem, which is nowadays accepted as the relevant model to describe micromagnetic phenomena. However, micromagnetics is one prototype of a non-convex and non-local multiscale problem and, from a numerical point of view, thus hardly accessible.

In [3], a reduced model for thin-film micromagnetics has been introduced which is consistent with the prior works [1] and [5]. Let $\omega \subseteq \mathbb{R}^2$ denote a bounded Lipschitz domain with diameter $\ell = 1$. This domain represents our ferromagnetic sample $\Omega = \omega \times [0,t]$, whose thickness $t > 0$ is neglected for simplicity. Here, we consider a uniaxial material with in-plane easy axis $\mathbf{e}_1$. With an applied exterior field $\mathbf{f} : \omega \to \mathbb{R}^2$, we seek a minimizer $\mathbf{m}^*$ of the reduced energy

$$e(\mathbf{m}) = \frac{1}{2}\int_{\mathbb{R}^3} |\nabla u|^2\,dx + \frac{q}{2}\int_\omega \mathbf{m}_2^2\,dx - \int_\omega \mathbf{f}\cdot\mathbf{m}\,dx \tag{1}$$

with the convex side constraint $|\mathbf{m}| \leq 1$. The magnetostatic potential $u : \mathbb{R}^3 \to \mathbb{R}$ is related to the magnetization via

$$\int_{\mathbb{R}^3} \nabla u \cdot \nabla\varphi\,dx = \int_\omega \mathbf{m}\cdot\nabla\varphi(x,0)\,dx \quad \text{for all} \quad \varphi \in \mathscr{D}(\mathbb{R}^3). \tag{2}$$

In contrast to [2] and [4], where the focus is on a distributional point of view, we give a precise and appropriate functional analytic framework in a certain subspace of $H^{1/2}(\mathrm{div},\omega) := \{\mathbf{m} \in L^2(\omega)^2 \,|\, \nabla\cdot\mathbf{m} \in \widetilde{H}^{-1/2}(\omega)\}$. Existence and uniqueness of a minimizer $\mathbf{m}^*$ in our functional setting is proven. We propose a numerical discretization strategy by use of lowest-order Raviart-Thomas finite elements and provide a priori error estimates. First numerical examples conclude the talk.

[1] P. Bryant and H. Suhl: Thin-film patterns in an external field. *Appl. Phys. Lett.* **54** (1989), 2224–2226.

[2] A. DeSimone, R. V. Kohn, S. Müller, and F. Otto: A Reduced Theory for Thin-Film Micromagnetics. *Comm. Pure Appl. Math.* **Vol. LV** (2002), 1408–1460.

[3] A. DeSimone, R. V. Kohn, S. Müller, F. Otto, and R. Schäfer: Two-dimensional modeling of soft ferromagnetic films. *Proc. R. Soc. Lond.* **A 457** (2001), 2983–2991.

[4] J. Drwenski: Numerical methods for a reduced model in thin-flim micromagnetics, Dissertation, University of Bonn, 2008.

[5] H. A. M. van den Berg: Self-consistent domain theory in soft-ferromagnetic media. II. Basic domain structures in thin film objects. *J. Appl. Phys.* **60** (1986), 1104–1113.