A determinant identity for Laplace matrices, counting spanning trees, and reconstructing electrical networks

TUE/P2
11:30–11:50

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Using an determinant identity of Sylvester we show that every minor of a Laplace matrix can be written in terms of those minors, where two rows and the corresponding two columns are deleted.

This identity can be applied to the enumeration of spanning trees. Specifically, we show that if a subgraph of a graph G is replaced by an electrically equivalent graph, the number of spanning trees only changes by a factor that does not depend on G. This allows us to employ techniques from the theory of electrical networks—such as the Wye-Delta transform—to determine the number of spanning trees of a graph.

Furthermore, the identity also leads to a solution of the following inverse problem: Given an electrical network, the so-called effective resistance between any two vertices can be computed in terms of resistances. Our identity yields an explicit formula for resistances in terms of effective resistances.

Joint work with Stephan Wagner.