Fractal tiles associated with shift radix systems. I

THU/EPCOS
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For each vector $\mathbf{r} \in \mathbb{R}^d$, define the mapping $\tau_{\mathbf{r}} : \mathbb{Z}^d \to \mathbb{Z}^d$ by

$$\mathbf{z} = (z_1, \ldots, z_d) \mapsto (z_2, \ldots, z_d, -\lfloor \mathbf{rz} \rfloor).$$

The dynamical system $(\mathbb{Z}^d, \tau_{\mathbf{r}})$ is called a *shift radix system* (SRS for short). SRS are strongly related to beta-numeration, canonical number systems and numeration with respect to rational bases. The present talk is devoted to geometric properties of SRS.

Indeed, one can attach to each SRS $(\mathbb{Z}^d, \tau_{\mathbf{r}})$, which is contractive in a certain sense, a family of tiles in a natural way. We will give basic properties of this family of tiles. For instance, we show that it consists of compact sets which admit a covering of $\mathbb{R}^d$. For large classes of parameters $\mathbf{r}$, we are even able to show that this family forms a (multiple) tiling of $\mathbb{R}^d$. In general, this tiling consists of infinitely many different shapes of tiles. Moreover, the tiles are not necessarily solutions of graph-directed function systems, as it is the case for many known notions of tiles related to numeration.

Besides that, these tiles admit new tilings associated with beta-numeration with respect to non-unit Pisot numbers, as well as with canonical number systems with respect to non-monic polynomials.

The second part of this talk will be given by W. Steiner.