

q -umbral calculus, the key to q -calculus

Thomas Ernst (Univ. Uppsala)

MON/P2 17:30–17:50

We first outline the foundations of the authors q -umbral calculus, which consists of an infinite alphabet A of letters or umbrae, with two dual q -additions, the Nalli-Ward-Alsalam (NWA) q -addition and the Jackson-Hahn-Cigler (JHC) q -addition. The Carlitz-Gould operator can be described by the JHC, and the NWA is both commutative and associative. The tilde operator makes a connection between q -hypergeometric functions, Γ_q -functions and Jacobi elliptic functions possible. The NWA decides the convergence region of 50% of the q -Appell- and q -Lauricella functions, and implies the possibility of certain q -complex numbers with similar properties as complex numbers. We can also introduce matrix pseudo-groups as the q -exponential function of ordinary matrix Lie algebras. This new kind of matrix Lie groups has two multiplications, corresponding to NWA and JHC, supplied with an associative structure.

- [1] P. APPELL, J. KAMPÉ DE FÉRIET: *Fonctions hypergéométriques et hypersphériques*. Paris 1926.
- [2] J. CIGLER: Operatormethoden für q -Identitäten. *Monatsh. Math.* **88** (1979), 87–105.
- [3] W. HAHN: Beiträge zur Theorie der Heineschen Reihen. *Mathematische Nachrichten.* **2** (1949), 340–379.
- [4] G.-C. ROTA, B. D. TAYLOR: The classical umbral calculus. *SIAM J. Math. Anal.* **25** (1994), no. 2, 694–711.
- [5] M. WARD: A calculus of sequences. *Amer. J. Math.* **58** (1936), 255–266.