Quadrics of Revolution Through Given Points

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The problem of determining all cylinders of revolution through 4 given points \( P_i \) has been solved by H. Schaal [1, 2]: There is a 1-parametric set of such cylinders and in case of non-coplanar points \( P_i \) their axes fulfill an algebraic surface of degree 3. The task of determining all cylinders of revolution through 5 points is an algebraic problem of order 6: In the generic case there exist 6 solution cylinders at most [5]. The cones of revolution through 4 given points establish a 2-parametric set, the locus of vertices is an algebraic surface of order 14, see [3,4].

In my presentation I introduce a unified approach to identifying different kinds of quadrics of revolution through a given number \( n \) of points. With this method it can be easily seen that the axes of the 1-parametric set of quadrics of revolution through 6 prescribed points intersect the plane of infinity along a conic section. Moreover I will show that there are at most 4 quadrics of revolution through 7 given points and at most 12 cones of revolution through 6 given points in the generic case. The fore-mentioned task of determining the cylinders of revolution through 5 points can also be treated by the introduced method. In each case I can give examples where the maximal solution numbers 4, 12 and 6 are obtained by real quadrics, cones and cylinders of revolution, respectively.