Quadrics of Revolution Through Given Points

MON/BE01
17:30–17:50

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The problem of determining all cylinders of revolution through 4 given points P_i has been solved by H. SCHAAL [1, 2]: There is a 1-parametric set of such cylinders and in case of non-coplanar points P_i their axes fulfill an algebraic surface of degree 3. The task of determining all cylinders of revolution through 5 points is an algebraic problem of order 6: In the generic case there exist 6 solution cylinders at most [5]. The cones of revolution through 4 given points establish a 2-parametric set, the locus of vertices is an algebraic surface of order 14, see [3,4].

In my presentation I introduce a unified approach to identifying different kinds of quadrics of revolution through a given number n of points. With this method it can be easily seen that the axes of the 1-parametric set of quadrics of revolution through 6 prescribed points intersect the plane of infinity along a conic section. Moreover I will show that there are at most 4 quadrics of revolution through 7 given points and at most 12 cones of revolution through 6 given points in the generic case. The fore-mentioned task of determining the cylinders of revolution through 5 points can also be treated by the introduced method. In each case I can give examples where the maximal solution numbers 4, 12 and 6 are obtained by *real* quadrics, cones and cylinders of revolution, respectively.

[1] H. SCHAAL: Ein geometrisches Problem der metrischen Getriebesynthese. *Sb. d. Österr. Akad. d. Wiss., math.-nat. Klasse, Abt II* **194**, Heft 1–3 (1985), 39–53.

[2] H. SCHAAL: Konstruktion der Drehzylinder durch vier Punkte einer Ebene. *Sb. d. Österr. Akad. d. Wiss., math.-nat. Klasse, Abt II* **195**, Heft 4–7 (1986), 406–418.

[3] U. STROBEL: Über die Drehkegel durch vier Punkte. *Sb. d. Österr. Akad. d. Wiss., math.-nat. Klasse* **198** (1989), 281–293.

[4] U. STROBEL: Über die Drehkegel durch vier Punkte. Teil II. *Sb. d. Österr. Akad. d. Wiss., math.-nat. Klasse* **200** (1991), 91–109.

[5] P.J. ZSOMBOR-MURRAY, S. EL FASHNY: A cylinder of revolution on five points, *Journal for Geometry & Graphics* **10**, 2 (2007), 125–131.