Exceptional Sequences of Line Bundles on $\mathbb{C}*$-Surfaces

THU/110
12:00–12:20

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Continuing the previous talk by N. Ilten, I will present further results from [1].

First, I shall recall some results from L. Hille and M. Perling in [2]. From a full exceptional sequence of line bundles $\mathscr{E}$ on a rational surface X they develop the slightly more general notion of a *toric system* $\mathscr{A}$ and show how we can associate to such a sequence of line bundles a toric variety $\mathrm{TV}(\mathscr{A})$.

I will focus on the behaviour of toric systems under degeneration. It was loosely conjectured by A. Bondal that for an exceptional sequence $\mathscr{E}$ the step from X to $\mathrm{TV}(\mathscr{E})$ has something to do with degeneration. Our following result makes this more concrete for rational $\mathbb{C}^*$-surfaces:

THEOREM. *Consider a homogeneous degeneration from a rational $\mathbb{C}^*$-surface X_s to another rational $\mathbb{C}^*$-surface X_0 along with an toric system $\mathscr{A}_s$ on X_s. Then the induced degeneration of $\mathscr{A}_s$ is again a toric system $\mathscr{A}_0$ on X_0. Moreover,* $\mathrm{TV}(\mathscr{A}_s) = \mathrm{TV}(\mathscr{A}_0)$.

Another technique from [2] is called *augmentation*, which, given a blowup X' of X, produces toric systems on X' from toric systems on X. We obtain exceptional sequences $\mathscr{E}$ on any rational surface X by successively augmenting an exceptional sequence on a Hirzebruch surface. In general the degeneration of such an $\mathscr{E}$ isn't necessarily exceptional. In light of this, we define the notion of *compatibility*, which can be checked recursively, and show

THEOREM. *Consider a homogeneous degeneration of rational $\mathbb{C}^*$-surfaces from X to Y and $\mathscr{E}$ an exceptional sequence on X. Then $\mathscr{E}$ is compatible with this degeneration if and only if its degeneration is an augmented exceptional sequence.*

[1] A. HOCHENEGGER AND N. ILTEN: *Families of Divisors on T-Varieties and Exceptional Sequences on $\mathbb{C}^*$-Surfaces.* arXiv:0906.4292v1 [math.AG], 2009.

[2] L. HILLE AND M. PERLING: *Exceptional Sequences of Invertible Sheaves on Rational Surfaces.* arXiv:0810.1936 [math.AG], 2008.