A closer look at BDF schemes and their stability

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Backward differentiation (BDF) schemes of lower orders are well-known for their favorable stability properties like strong $A(\alpha)$-stability. Even for the scalar stiff model $y' = \lambda y$, however, sharp stability estimates are not straightforward due to the presence of parasitic roots of the characteristic equation. In particular, the distribution (separation) of these roots is not obvious in the region near the boundary of the stability domain, i.e., for mildly stiff situations (typically: small stepsize h together with moderate degree of stiffness). Since BDF schemes are widely used, a precise description of the spectrum is of interest.

In [1], the location of parasitic roots as compared to the principal root was studied, and it was shown that for orders up to 5, there is a uniform 'gap' between these roots in the mildly stiff case, which enables quantitative estimates for the worst-case propagation of perturbations for an arbitrary degree of stiffness. (For larger stepsizes such an estimate is straightforward due to strong stability.) For the BDF 2 method this analysis is rather simple. It shows that, for uniform stepsizes, BDF 2 is comparable to backward Euler in this respect. For the higher order schemes, the uniform, quantitative estimation of the location of parasitic roots requires a rather intricate analysis based on bivariate polynomial algebra and techniques from complex analysis.

These results can also be used for a more precise characterization of the stability of BDF schemes for general linear evolution equations, and as a tool in the derivation of a non-classical, perturbed asymptotic expansion of the global error. In particular, in [2] such a result is derived for the BDF 2 scheme applied to an abstract parabolic problem, assuming a a sufficiently smooth solution. The analysis is based on discrete resolvent calculus and G-stability. Under quite general assumptions, we obtain a representation for the global error e_ν of the form

$$e_\nu = h^2 e_2(t_\nu) + h^3 e_3(t_\nu) + \omega_\nu \qquad (t_\nu = \nu h).$$

Here, $e_2(t)$ and $e_3(t)$ are smooth, h-independent functions. The remainder ω_ν behaves like $\mathscr{O}(h^3)$ at the first grid points but is quickly damped out with increasing ν. This closely resembles the behavior of the backward Euler scheme studied in earlier work.

[1] W. AUZINGER, W. HERFORT: A uniform quantitative stiff stability estimate for BDF schemes, *Opuscula Mathematica* **26/2** (2006), 203-227.

[2] W. AUZINGER, F. KRAMER: On the stability and error structure of BDF schemes applied to linear parabolic evolution equations, submitted.