The Degree Distribution of Unlabelled Outerplanar Graphs

TUE/P2
16:00–16:20

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An outerplanar graph is a planar graph which has an embedding in the plane such that all nodes lie on the outer face. We focus on the degree distribution in unlabelled outerplanar graphs. The basis for our result are works by Fusy et al. [1], who asymptotically enumerated unlabelled outerplanar graphs, and by Drmota, Giménez and Noy [3] and McDiarmid, Steger and Welsh [4] who obtained results on the degree distribution in the labelled case. Similarly as in the papers mentioned above, we use a generating functions approach and methods of singularity analysis. To deal with unlabelled structures, we have to take into account all automorphisms evolving from permutations on the set of vertices, using cycle index sums and Pólya's theory. With the help of the mentioned tools, we prove the following theorem:

THEOREM 1. *For $k \geq 2$, let X_n^k denote the number of vertices of degree k in an unlabelled 2-connected outerplanar graph with $n+2$ vertices. Then X_n^k satisfies a central limit theorem with expected value $\mathbb{E}X_n^k \sim \mu_k \cdot n$ with $\mu_k > 0$ and variance $\mathbb{V}X_n^k \sim \sigma_k^2 \cdot n$.*

Interestingly, this result, even the expected value μ_k, is identical with the one for 2-connected labelled outerplanar graphs, which implies that these graphs are quite rigid and thus the (few) appearing symmetries do not influence the asymptotic behaviour. In the connected case there is an exponential number of symmetries which will influence the result, thus the expected value will differ from the one in the labelled case, but still a central limit law can be obtained.

[1] M.BODIRSKY, E.FUSY, M.KANG AND S.VIGERSKE: *Enumeration of Unlabeled Outerplanar Graphs*. Electronic Journal of Combinatorics, 14 (2007), no. 1, research paper 66.
[2] M.DRMOTA: *Random Trees*. Springer Verlag Wien, 2009.
[3] M.DRMOTA, O.GIMÉNEZ AND M.NOY: *Vertices of given degree in series-parallel graphs*. Random Structures Algorithms, to appear.
[4] C.MCDIARMID, A.STEGER AND D.J.A.WELSH: *Random Planar Graphs*. J. Combin. Theory Ser.B, 93:187–205. 2005.