An Interpolatory Estimate for the UMD-Valued Directional Haar Projection

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The Calculus of Variations, in particular the theory of compensated compactness has long been a source of hard problems in harmonic analysis. One developement started with the work of F. Murat and L. Tartar, where the decisive theorems were on Fourier multipliers of Hörmander type. Using time–frequency localizations relying on Littlewood–Paley and wavelet expansions, as well as modern Calderon–Zygmund theory S. Müller and J. Lee, P. F. X. Müller, S. Müller extended and strengthened the results obtained by Fourier multiplier methods.

Avoiding this time–frequency localization by utilizing T. Figiel's canonical martingale decomposition permits to further extend the results to the Bochner–Lebesgue space $L^p_X(\mathbb{R}^n)$, provided X satisfies the UMD–property. For $n \geq 2$, $1 < p < \infty$, $\varepsilon = (\varepsilon_1, \dots, \varepsilon_n) \in \{0,1\}^n$ and $u \in L^p_X(\mathbb{R}^n)$ define the directional Haar projection $P^{(\varepsilon)}u = \sum_{Q\in\mathscr{Q}} \langle u, h_Q^{(\varepsilon)}\rangle\, h_Q^{(\varepsilon)}\, |Q|^{-1}$, and let R_i, $1 \leq i \leq n$ denote the Riesz-transform acting on the i–th coordinate. If $\varepsilon_{i_0} = 1$ and L^p_X has type $\mathscr{T}(L^p_X)$, the main result of this paper is the interpolatory estimate

$$\|P^{(\varepsilon)}u\|_{L^p_X} \leq C\, \|u\|_{L^p_X}^{1/\mathscr{T}(L^p_X)} \cdot \|R_{i_0}u\|_{L^p_X}^{1-1/\mathscr{T}(L^p_X)},$$

holding true for all $u \in L^p_X$, where $C > 0$ depends only on n, p, X and $\mathscr{T}(L^p_X)$.