On Pollard's Theorem for General Abelian Groups

TUE/P2
17:00–17:20

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Let $t \geq 1$, let A and B be finite, nonempty subsets of an abelian group G, and let $A +_i B$ denote all the elements c with at least i representations of the form $c = a + b$ with $a \in A$ and $b \in B$. For $|A|, |B| \geq t$, we show that either

$$\textstyle\sum_{i=1}^{t} |A +_i B| \geq t|A| + t|B| - 2t^2 + 1, \tag{1}$$

or else there exist $A' \subseteq A$ and $B' \subseteq B$ with

$$\begin{aligned} l &:= |A \setminus A'| + |B \setminus B'| \leq t - 1, \\ A' +_t B' &= A' + B' = A +_t B, \text{ and} \\ \textstyle\sum_{i=1}^{t} |A +_i B| &\geq t|A| + t|B| - t|H|, \end{aligned} \tag{2}$$

where $H := \{h \in G \mid h + A +_t B = A +_t B\}$ is the (nontrivial) stabilizer of $A +_t B$. (Actually, a more accurate bound is obtained in place of (2), but we here only display its simplified consequence). This gives a version of Pollard's Theorem [1][2] for general abelian groups in the tradition of Kneser's Theorem [1]. The proof makes use of additive energy and other recent advances in employing the Dyson transform [1][3]. Examples are given that show that such a Kneser-type result cannot hold when the bound in (1) is extended to the original bound of Pollard (for $t \geq 3$), and that reduction present in (1) is of the correct order of magnitude (quadratic in t). However, in the case $t = 2$, we improve (1) to

$$|A +_1 B| + |A +_2 B| \geq 2|A| + 2|B| - 4.$$

[1] M. NATHANSON: *Additive Number Theory: Inverse Problems and The Geometry of Sumsets*. Graduate Texts in Matmematics **165**. Springer-Verlag, New York, 1996.

[2] J. M. POLLARD: A Generalisation of the Theorem of Cauchy and Davenport. *J. London Math. Soc.* **8** (1974), 460–462.

[3] T. TAO AND V. VU: *Additive Combinatorics* . Cambridge Studies in Advanced Mathematics **105**. Cambridge University Press, Cambridge, 2006.