A new homology for infinite graphs and metric continua

Agelos Georgakopoulos (TU Graz)

Tue/P2
18:00–18:20

Motivated by recent research on the homology of infinite graphs compactified by their ends, we introduce a new homology for arbitrary metric continua. This development triggers interactions between graph theory and other fields.

The first simplicial homology group of a graph G is known in graph theory as the *cycle space* $\mathscr{C}(G)$ *of* G. For finite graphs, the cycle space is well-studied and many results are known; see [2] for instance. In infinite graphs, however, several of these results become false. To remedy this, Diestel and Kühn [3] proposed a new homology, combining ideas from graph theory and algebraic topology. This *topological cycle space* $\mathscr{C}(G)$ has been surprisingly successful; indeed, various authors have shown that all the standard properties of the cycle space in a finite graph generalise to infinite graphs if the topological cycle space is used, see [1,2] for more details.

One of the most fundamental facts about the cycle space of a finite graph is the following:

Theorem 1. *([1]) Every element of* $\mathscr{C}(G)$ *has a representative which is a union of edge-disjoint cycles.*

Theorem 1 was generalised by Diestel and Kühn to the topological cycle space of a locally finite graph, yielding a strong theorem with many applications, see [4]. At first sight it seems that Theorem 1 is not relevant to the homology of spaces other than graphs, because of the explicit mention of edges. However, this is less so if one considers the following (equivalent) reformulation of Theorem 1:

Assertion 2. *Every element of* $\mathscr{C}(G)$ *has a representative of minimal length.*

The classical first singular homology group of a metric continuum fails to satisfy Assertion 2. We show how to construct a new homology-like theory that applied to graphs yields the aforementioned topological cycle space and allows the extention of Assertion 2 to arbitrary metric continua. This triggers further conjectures in the intersection of graph theory and algebraic topology. See [5] for details.

[1] R. Diestel. The cycle space of an infinite graph. *Comb., Probab. Comput.*, 14:59–79, 2005.
[2] R. Diestel. *Graph Theory* (3rd edition). Springer-Verlag, 2005.
[3] R. Diestel and D. Kühn. Topological paths, cycles and spanning trees in infinite graphs. *Europ. J. Combinatorics*, 25:835–862, 2004.
[4] A. Georgakopoulos. Topological circles and Euler tours in locally finite graphs. 16:#R40, 2009.
[5] A. Georgakopoulos. Graph topologies induced by edge lengths. arXiv.org 0903.1744, submitted.