A Line-Based Discretization of Curvature Lines

TUE/EPCOS
16:30–16:50

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A well-known discretization of the net of curvature lines on a smooth surface is a *circular net* – a map $f\colon \mathbb{Z}^2 \to \mathbb{R}^3$ such that any four points f_{ij}, $f_{i+1,j}$, $f_{i,j+1}$, and $f_{i+1,j+1}$ lie on a circle [1, Chapter 3]. A different discretization, *conical nets* [1, Chapter 3; 3], have planar quads such that the planes at any vertex are tangent to a cone of revolution. The characteristic property shared by curvature lines, circular nets, and conical nets is the concurrency of neighboring normals (surface normals, circle axes, and cone axes, respectively). Circular nets may be seen as a "point-based" discretization of curvature lines while conical nets are "face-based."

In our talk we present a discretization of one family of curvature line by an oriented line congruence – a map from $\mathbb{Z}^2$ to the space of oriented lines in $\mathbb{R}^3$ such that neighboring lines intersect. The characteristic property is that the lines of an elementary quadrilateral lie on a hyperboloid of revolution.

Among the numereous properties of these congruences we mention just a few:

- The hyperboloid axes can be seen as a discrete surface normals; neighboring axes are concurrent.
- There exists a family of congruences of the same type such that corresponding lines are parallel and at constant distance (*offset congruences*). In contrast to circular and conical nets, this property is *not characteristic*.
- There exists a family of congruences of the same type such that corresponding lines meet orthogonally (*orthogonal congruences* – a discretization of the second family of curvature lines.)
- The edge-offset meshes of [2] are obtained as the limiting case of "flat" hyperboloids.
- Circular nets, conical nets and HR-congruences can be constructed from each other such that corresponding elements are incident.

[1] A. I. BOBENKO, YU. B. SURIS: *Discrete Differential Geometry: Integrable Structure.* Graduate Studies in Mathematics, Vol. 98, AMS 2008.

[2] H. POTTMANN, Y. LIU, J. WALLNER, A. I. BOBENKO, W. WANG: *Geometry of multi-layer freeform structures for architecture.* ACM Trans. Graph. **26**(3), #65, 1–11 (2007)

[3] H. POTTMANN, J. WALLNER: *The focal geometry of circular and conical meshes.* Adv. Comp. Math, **29** (2008), 249–268.