Integral Cayley graphs over abelian groups

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Let Γ be a finite, additive group, $S \subseteq \Gamma,\ 0 \notin S,\ -S = \{-s:\ s \in S\} = S$. The undirected *Cayley graph* $G = \mathrm{Cay}(\Gamma, S)$ over Γ with *connection set* S has vertex set $V(G) = \Gamma$ and edge set $E(G) = \{\{a,b\} : a - b \in S\}$. The eigenvalues of a graph G are the eigenvalues of an arbitrary adjacency matrix of G. A graph is called *integral*, if all of its eigenvalues are integers. We say that the group Γ is *Cayley integral*, if every undirected Cayley graph over Γ is integral.

For an abelian group Γ we show that $\mathrm{Cay}(\Gamma, S)$ is integral, if $S \in B(\Gamma)$, where $B(\Gamma)$ denotes the boolean algebra generated by the subgroups of Γ. For cyclic groups the converse is also true.

We show that all nontrivial, abelian, Cayley integral groups are represented by

$$Z_2^n,\ Z_3^n,\ Z_4^n,\ Z_2^m \oplus Z_3^n,\ Z_2^m \oplus Z_4^n,\ \ m,\ n \geq 1.$$

Here Z_k denotes the cyclic group of integers modulo k.

The *Hamming graph* $H = Ham(m_1, \ldots, m_r; D)$ has as its vertex set $V = Z_{m_1} \oplus \ldots \oplus Z_{m_r}$. The set $D = \{d_1, \ldots, d_k\}$ consists of integers d_i, $1 \leq d_1 < d_2 < \ldots < d_k \leq r$. Vertices $x \neq y$ are adjacent in H, if their *Hamming distance* is in D. All Hamming graphs turn out to be integral Cayley graphs.

We like to mention that the graph associated with the popular Sudoku puzzle for arbitrary $n^2 \times n^2$-format is an integral Cayley graph.