Discrete Sobolev-Poincaré inequalities for finite volume Voronoi approximations

MON/E3.1
15:30–15:50

Annegret Glitzky (Weierstrass Inst. Applied Analysis and Stochastics, Berlin)

In this talk we study discrete Sobolev inequalities. In the continuous situation the Sobolev imbedding estimates

$$\|u\|_{L^q(\Omega)} \le c_q \|u\|_{H^1(\Omega)} \quad \forall u \in H^1(\Omega)$$

for $q \in [1,\infty)$ in two space dimensions and for $q \in [1,6]$ in three space dimensions are well known [1].

For the finite volume discretized situation up to now a discrete analogon of these inequalities is available in the case of Dirichlet boundary conditions only. The 2d case for admissible finite volume meshes is treated in [3], Lemma 9.5. The corresponding 3d result is proven in [2], Lemma 1. But the techniques used there fail to work in the case of Neumann boundary conditions.

We prove a discrete Sobolev inequality for Neumann boundary conditions and finite volume Voronoi meshes. We use Sobolev's integral representation and estimate weakly singular integrals in the context of finite volumes. We establish the result for star shaped polyhedral domains and generalize it to the finite union of overlapping star shaped domains.

[1] R. A. ADAMS: *Sobolev spaces*, Academic Press 1975.

[2] Y. COUDIÈRE, T. GALLOUËT, AND R. HERBIN: Discrete Sobolev Inequalities and L^p error estimates for approximate finite volume solutions of convection diffusion equations, *M2AN* **35** (2001), 31–63.

[3] R. EYMARD, T. GALLOUËT, AND R. HERBIN: The finite volume method. In: *Handbook of Numerical Analysis VII* (Ph. Ciarlet and J. L. Lions, eds.), North Holland, 2000, pp. 723–1020.