Convergence of adaptive BEM

FRI/P2
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A posteriori error estimators and adaptive mesh-refinement have themselves proven to be an important tool for scientific computing. For error control in finite element methods (FEM), there is a broad variety of a posteriori error estimators available, and convergence as well as optimality of adaptive FEM is well-studied in the literature. This is in sharp contrast to the boundary element method (BEM). Although a posteriori error estimators and adaptive algorithms are also successfully applied to boundary element schemes, even convergence of adaptive BEM is hardly understood mathematically. In our contribution, we present and discuss recent mathematical results [1,3] which give first positive answers for adaptive BEM.

As BEM model problem for our talk serves the weakly-singular integral equation

$$Vu = f$$

associated with the Laplace operator in 2D and 3D and stated in the energy space $\widetilde{H}^{-1/2}(\Gamma)$. We use the lowest-order Galerkin method with piecewise constant ansatz and test functions and consider standard adaptive algorithms of the type

$$\boxed{\textit{Solve}} \longrightarrow \boxed{\textit{Estimate}} \longrightarrow \boxed{\textit{Mark}} \longrightarrow \boxed{\textit{Refine}}$$

It is a simple consequence of functional analysis that the sequence u_ℓ of Galerkin solutions generated by this algorithm, tends to some limit $u_\infty \in \widetilde{H}^{-1/2}(\Gamma)$. It is, however, a priori unknown whether u_∞ coincides with the unique exact solution $u \in \widetilde{H}^{-1/2}(\Gamma)$ of the integral equation.

For a posteriori error estimation, we use certain $(h-h/2)$-type error estimators μ_ℓ from [4], and element marking is done by the ℓ_2-criterion introduced by Dörfler [2]. We then treat the convergence

$$\lim_{\ell\to\infty} u_\ell = u \quad \text{as well as} \quad \lim_{\ell\to\infty} \mu_\ell = 0$$

for both, isotropic and anisotropic mesh-refinement.

[1] M. AURADA, S. FERRAZ-LEITE, D. PRAETORIUS: A new notion of convergence for adaptive boundary element methods. Work in progress 2009.

[2] W. DÖRFLER: A convergent adaptive algorithm for Poisson's equation. *SIAM J. Numer. Anal.* **33**, (1996), 1106–1124.

[3] S. FERRAZ-LEITE, C. ORTNER, D. PRAETORIUS: Convergence of simple adaptive Galerkin schemes based on h-h/2 error estimators. Submitted for publication 2009.

[4] S. FERRAZ-LEITE, D. PRAETORIUS: Simple a posteriori error estimators for the h-version of the boundary element method. *Computing* **83**, (2008), 135–162.