On the cohomology of arithmetic subgroups of certain rational forms of G_2

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We investigate the (co)homology of arithmetic subgroups of the exceptional group G_2. Therefore, we introduce a geometric construction of cycles and present a formula to compute intersection numbers of these. We realize a rational form of G_2 as group of automorphisms of a certain octonion algebra. After that we can define certain reductive subgroups which give rise to these geometric cycles. Then we will compute their intersection numbers. In the case of G_2, the description of the contribution of cycles to the homology is "complete"in the following sense: first, in every cohomological degree with non-zero $(\mathfrak{g}, K)$-cohomology, there is a non trivial cycle and second, we use at least one group of every possible type of reductive subgroups of G_2 to construct a cycle. Further, all Poincaré dual classes of these cycles are non invariant under G_2; hence they are related to some unitary representations. All that can also be done in the case of local coefficients. Here one has to find in a finite dimensional irreducible representation of G_2 certain vectors. This can be done by applying the Schur functor to the space of pure octonions.