The sum of digits of squares in the Gaussian Integers

THU/EPCOS
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Inspired by recent results of Mauduit and Rivat [1] and of Drmota, Rivat and Stoll [2] we study the complex sum-of-digits function s_q for squares with respect to a base q of a canonical number system in the Gaussian integers $\mathbb{Z}[i]$. In particular, we show that the sequence $\left(\alpha s_q\left(n^2\right)\right)_{n\in\mathbb{Z}[i]}$ is uniformly distributed modulo 1 if and only if $\alpha \in \mathbb{R}\setminus\mathbb{Q}$. Furthermore we introduce special sets of Gaussian integers (related to Følner sequences) for which we can determine the order of magnitude of the number of integers z for which $s_q\left(z^2\right)$ lies in a fixed residue class $\bmod m$. We prove also a local limit theorem for the sum-of-digits function of squares. For example, we can provide asymptotic expansions for the numbers $\#\left\{n\in C_N : s_q\left(n^2\right)=k\right\}$ where C_N is a segment of a disc with angle γ and radius $\sqrt{N}$. Estimates of exponential sums, the study of Fourier transforms and probabilistic methods play a crucial role in proving these results.

[1] C. MAUDUIT AND J. RIVAT: *La Somme des Chiffres des Carrés*. Acta Mathematica, to appear
[2] M. DRMOTA, J. RIVAT AND T. STOLL: *The sum of digits of primes in* $\mathbb{Z}[i]$. Monatshefte für Mathematik, 2008