How hard is it to find extreme Nash equilibria in network congestion games?

TUE/AE01
12:00–12:20

*Elisabeth Gassner** (TU Graz), *Johannes Hatzl* (TU Graz), *Sven O. Krumke* (Univ. Kaiserslautern), *Heike Sperber* (Univ. Kaiserslautern), *Gerhard J. Woeginger* (TU Eindhoven)

We study the complexity of finding extreme pure Nash equilibria in symmetric (unweighted) network congestion games: There are N users each routing the same amount of unsplittable flow from a single source s to a single sink t through a directed graph $G = (V, E)$. The edges of G are equipped with non-decreasing latency functions $\ell_e : \mathbb{N}_0 \to \mathbb{R}_0^+$ for all $e \in E$ modelling the congestion effects. Each users chooses a path P. Assume that a user takes path P. Then his latency is equal to the sum of the latencies on the edges of P while the makespan is the maximum of the user's latencies. Finally, a Nash equilibrium is a stable situation in which no user wants to deviate from his chosen path because he cannot decrease his experienced latency this way.

The KP-Model named after Koutsoupias and Papadimitriou [1] describes the situation in which users of possibly different size assign their traffic to parallel links with linear latency functions. Fotakis et al. [2] came up with the question whether a best or worst pure equilibrium w.r.t. to makespan can be computed efficiently and established that in the KP-Model both problems are NP-hard. Gairing et al. [3] added that it is even hard to approximate the worst equilibrium social cost on identical links while there is a PTAS for the best equilibrium social cost.

The hardness proofs for extreme equilibria stated above are based on the users' different sizes and the close relationship to scheduling and bin-packing problems. The question arises whether finding extreme Nash equilibria for unit-size users is substantially easier as for the unit-size case the corresponding scheduling and bin-packing instances become polynomially solvable.

We consider the unit-size case and show that on series-parallel graphs a worst Nash equilibrium can be found by a Greedy approach while finding a best equilibrium is NP-hard. For a fixed number of users we give a pseudo-polynomial algorithm to find the best equilibrium in series-parallel networks. For general network topologies also finding a worst equilibrium is NP-hard.

[1] E. KOUTSOUPIAS, C. PAPADIMITRIOU: Worst-case equilibria. *Lecture Notes in Computer Science* **1563** (1999), 404–413.

[2] D. FOTAKIS, S. KONTOGIANNIS, E. KOUTSOUPIAS, M. MOVRANICOLAS, P. SPIRAKIS: The structure and complexity of Nash equilibria for selfish routing games. In: *Proceedings of the 29th International Colloquium on Automata, Languages and Programming.* Springer-Verlag, 2002, pp. 123–134.

[3] M. GAIRING, T. LÜCKING, M. MAVRONICOLAS, B. MONIEN, P. SPIRAKIS: The structure and complexity of extreme Nash equilibria. *Theoretical Computer Science* **343** (2005), 133–157.