

Random Walks on Directed Covers of Graphs

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Suppose we are given a connected, directed graph G with vertex set V , edge set E (without multiple edges), and root i_0 . We construct a labelled tree \mathcal{T} from G as follows: the root is labelled with i_0 ; recursively, if x is a vertex in the tree with label $i \in V$, then x has a successor with label j if and only if there is an edge from i to j in G . The tree \mathcal{T} is called the *directed cover* of G , or also known as *periodic tree*. We consider a nearest neighbour random walk on \mathcal{T} which arises in a natural way from a nearest neighbour random walk on G .

We expand the existing theory of directed covers of finite graphs to those of infinite graphs. We give a short comparison of behaviour in the finite setting, when G is finite, and in our more general setting of infinite G . This comparison includes the classification of recurrence/transience behaviour and (in)equality of upper and lower growth rate of \mathcal{T} . The main result is the proof of existence and positivity of the asymptotic entropy under reasonable assumptions. That is, we prove that there is a number $h > 0$ such that $h = \lim_{n \rightarrow \infty} n^{-1} \log \pi^{(n)}(X_n)$, where X_n is the random vertex at which the random walk is at time n and $\pi^{(n)}(\cdot)$ is the distribution of X_n . Moreover, our proof gives an explicit formula which is also a new result for directed covers of finite graphs.

- [1] T. NAGNIBEDA AND W. WOESS: Random walks on trees with finitely many cone types. *J. Theoret. Probab.*, 2002, pp. 399–438.
- [2] R. LYON AND Y. PERES: *Probability on Trees and Networks*. In preparation. Current version available at mypage.iu.edu/~rdlyons/, 2008.
- [3] I. BENJAMINI AND Y. PERES: Tree-indexed random walks on groups and first passage percolation. *Probab. Theory Related Fields*, 1994, pp. 91–112.