## Combinatorial properties of a general domination problem with parity constraints

TUE/P2
16:30–16:50

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We consider various properties of a general parity domination problem: given a graph $G$ on $n$ vertices, one is looking for a subset $S$ of the vertex set such that the open/closed neighborhood of each vertex contains an even/odd number of vertices in $S$ (it is prescribed individually for each vertex which of these applies). This problem is motivated by the following remarkable result of Sutner [3]:

THEOREM. *For every graph $G = (V,E)$, there exists a set $S \subseteq V$ such that $|N[v] \cap S|$ is odd for every $v \in V$.*

In this talk, we define the parameter $s(G)$ (see [2]) to be the number of solvable instances out of $4^n$ possibilities and study the properties of this parameter. Upper and lower bounds for general graphs and trees are given as well as a remarkable recurrence formula for rooted trees. Furthermore, we give explicit formulas in several special cases and investigate random graphs.
Finally, we discuss the problem of finding such a set $S$ with minimum cardinality. It is well known that this problem is $\mathcal{NP}$-hard in general. However, it can be solved in linear time on graphs with bounded tree width and distance-hereditary graphs (see [1]).

[1] E. GASSNER, J. HATZL: A parity domination problem in graphs with bounded tree width and distance-hereditary graphs. *Computing* **82** (2008), 171–187.
[2] J. HATZL, S. WAGNER: Combinatorial properties of a general domination problem with parity constraints. *Discrete Mathematics* **308** (2008), 6355–6357.
[3] K. SUTNER: Linear cellular automata and the Garden-of-Eden *Math. Intelligencer* **11** (1989), 49–53.