Stochastic Particle Approximation to the Keller-Segel model in 2D

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We construct an approximation to the measure valued, global in time solutions to the Keller-Segel model in 2D, based on systems of stochastic interacting particles. The advantage of our approach is that it reproduces the well-known dichotomy in the qualitative behavior of the system and, moreover, captures the solution even after possible blow-up events. We present a numerical method based on this approach, discuss the related technical issues and show some numerical results.

The first step toward the convergence analysis of our method consists of considering a regularized particle scheme and showing that, as the number of particles tends to infinity, one recovers the solution of the regularized Keller-Segel system. The proof is based on the BBGKY-approach known from classical kinetic theory.

The second step is to describe the limit when the regularization parameter tends to zero, which is technically much more involved and requires an application of the framework of time dependent measures with defects. Subsequently, we pass to the limit when the number of particles tends to infinity and show that the resulting object is compatible with the measure valued solutions of the Keller-Segel system, in the sense that solutions of the latter generate solutions of the former. However, from fundamental reasons it is impossible to make a rigorous statement about the equivalence of these two. Finally, we provide a detailed description of the dynamics of the particle system consisting of two particles only, and explain why the analysis of systems with three or more particles remains an open problem.