The Dirichlet problem for Willmore surfaces of revolution

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Boundary value problems for the "Willmore equation" are nonlinear and, in particular, frame invarint counterparts of the linear clamped plate equation. The corresponding energy functional involves curvature integrals over the unknown surface. The Willmore equation is of interest in mechanics, membrane physics and, in particular, in differential geometry. Quite far reaching results were achieved concerning closed surfaces.

As for boundary value problems, by far less is known, see e.g. [3,4]. The Willmore equation is quasilinear and of fourth order. Due to the lack of any form of general comparison principles, the derivation of classical a-priori-bounds in general situations seems at the moment to be out of reach. So, it is reasonable to start investigating symmetric situations like e.g. surfaces of revolution. Nevertheless, the geometry is two-dimensional and difficulties due to the interplay of the principal curvatures have to be overcome. Thanks to the symmetry assumptions it is possible to pass to suitable a-priori bounded minimising sequences for the Willmore functional and to show existence of a Willmore surface of revolution for any symmetric set of Dirichlet boundary data.

The lecture is based on the joint work [1,2] with A. Dall'Acqua, K. Deckelnick (Magdeburg), S. Fröhlich (Free University of Berlin), and F. Schieweck (Magdeburg).

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- [1] A. DALL'ACQUA, S. FRÖHLICH, H.-CH. GRUNAU, F. SCHIEWECK: Symmetric Willmore surfaces of revolution satisfying arbitrary Dirichlet boundary data. Preprint 24-2008, Universität Magdeburg, 2008.
- [2] A. DALL'ACQUA, K. DECKELNICK, H.-CH. GRUNAU: Classical solutions to the Dirichlet problem for Willmore surfaces of revolution. *Adv. Calc. Var.* **1** (2008), 379–397.
- [3] J.C.C. NITSCHE: Boundary value problems for variational integrals involving surface curvatures. *Quart. Appl. Math.* **51**, (1993) 363–387.
- [4] R. SCHÄTZLE: The Willmore boundary value problem. Preprint, Universität Tübingen, 2006.