Orders in algebraic number fields with half-factorial localizations

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The maximal order $\mathcal{O}_K$ of an algebraic number field is a Dedekind domain, and its arithmetic is completely determined by its Picard group $\mathrm{Pic}(\mathcal{O}_K)$. In particular, $\mathcal{O}_K$ is factorial if and only if its Picard group is trivial. In contrast, non-principal orders are not integrally closed, hence they are never factorial, and their arithmetic depends not only on their Picard group but also on the localizations at singular primes. A non-principal order $\mathcal{O}$ with $|\mathrm{Pic}(\mathcal{O})| \geq 3$ inherits many arithmetical properties from the maximal order. In contrast, only little is known about the arithmetic of non-principal orders whose Picard group has at most two elements, even if all localizations are half-factorial. In this case, using special saturated submonoids as tools, we are able to give a quite explicit description of various arithmetical invariants such as the elasticity $\rho(\mathcal{O})$, the minimum distance $\min\Delta(\mathcal{O})$, and the catenary degree $\mathrm{c}(\mathcal{O})$. In particular, we prove that $\rho(\mathcal{O}) \in \left\{1, \frac{3}{2}, 2\right\}$ and $\min\Delta(\mathcal{O}) \leq 1$.