Monotone Curves

TUE/110
10:30–10:50

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Let L be a family of K closed linear subspaces of ℓ_2, and $z_0 \in \ell_2$. Consider the sequence of projections $z_i = P_{k_i} z_{i-1}$, where P_k denotes the orthogonal projection on the k-th space in L. According to Amemiya and Ando, the orbit $\{z_i\}$ always converges weakly. If $K = 2$ the sequence of projections even converges in norm according to a classical result of von Neumann. If $K \geq 3$, this is known only under additional assumptions, for example, if the sequence $\{k_i\}$ is periodic.

We estimate the rate of convergence of products of projections on K finite dimensional or finite co-dimensional subspaces in ℓ_2. The current proof gives dependence of the estimate on both the number of the subspaces K and on their maximal finite dimension (or codimesion) n. Dropping the dependence on n would result in the norm-convergence of the orbit of a point under any sequence of projections on finitely many closed subspaces of ℓ_2.

In connection with projections we stumbled upon the following curious question. The intuitive answer to it is “obviously no”, but we do not know any really simple proof of this “obvious fact”. Here is the question:

Let e_1 and e_2 be two orthogonal vectors in the unit sphere of ℓ_2. Given a small $\varepsilon > 0$, does there exist a piecewise linear curve γ connecting e_1 with $(1-\varepsilon)e_2$, so that the distance from the origin decreases along γ and all segments of γ are parallel to at most, say, 5 different vectors?

Using the methods we developed for products of projections, we show that for small $\varepsilon > 0$ there is *no* such a curve.