St. Kathrein am Offenegg – Styria

Alp–Workshop

July 4. & 5. 2009

Spectral and probabilistic properties
of random walks on random graphs

Program of the Talks

Vadim Kaimanovich (Jacobs University Bremen): Sa, 12:10-12:55
Stochastic homogenization of graphs: case studies

The talk is devoted to a discussion of the notion of stochastic homogeneity for random graphs and its applications in concrete situations.

Matthias Keller (University of Jena): Sa, 15:30-15:50
Random trees and absolutely continuous spectrum

We address the question in which situations one can expect absolutely continuous spectrum of the Laplace operator on random trees. In the case of radially symme tric trees absolutely continuous spectrum is very rare. In fact the trees need t o be eventually periodic. The situation with random trees is very different. If one considers random trees which are only close to regular trees (or more genera l: close to certain trees of finitely many cone types) one gets pure absolutely continuous spectrum of the Laplace operator almost surely. (This is joint work w ith Daniel Lenz and Simone Warzel.)

Franz Lehner (Genf University): Su, 14:00-14:45
On the Eigenspaces of Lamplighter Random Walks and Percolation Clusters on Graphs

We show that the Plancherel measure of the lamplighter random walk on a graph coincides with the expected spectral measure of the absorbing random walk on the Bernoulli percolation clusters. In the subcritical regime the spectrum is pure point and we construct a complete orthonormal basis consisting of eigenfunctions, supported on the finite percolation clusters of the given graph. This generalizes results of Grigorchuk, Zuk, Dicks and Schick and may shed some new light on the Atiyah conjecture and some problems in mathematical physics.

Daniel Lenz (University of Jena): Su, 10:00-10:45

Amenability of Horocyclic Products of uniformly growing trees

An overview of results regarding the amenability and non-amenability of horocyclic products is given, which have been obtain by 'elementary methods'. While the typical behaviour of this family of graphs for the case of random trees (such as GW-trees) can be clearly distinguished from 'deterministic' trees, such as homogeneous or isotropic trees, the question remained open which of these two characteristic types would be the typical one for trees with quasi-periodic order. The answer is given for uniformly growing trees, such as those with finitely many cone-types.

Bernt Metzger (University Paris 13): Sa, 14:00-14:20

The discrete Gross-Pitaevskii model and condensation in the single particle ground state

In the context of a tight-binding approximation of the Gross-Pitaevskii energy functional with a random background potential we want to discuss in dependence on the interaction coupling constant a criteria when the Gross-Pitaevskii ground state and the single particle ground state coincide.

Peter Mörters (University of Bath): Su, 16:00-16:45

Simultaneous multifractal analysis of branching and visibility measure on a Galton-Watson tree

On the boundary of a Galton-Watson tree one can define the visibility measure by splitting mass equally between the children of each vertex, and the branching measure by splitting unit mass equally between all vertices in the nth generation and then letting n go to infinity. In this talk we address the question of a joint multifractal spectrum, i.e., we ask for the Hausdorff dimension of the boundary points which have an unusual local dimension for both these measures simultaneously. The resulting two-parameter spectrum exhibits a number of surprising new features, among them the emergence of a swallowtail shaped spectrum for the visibility measure in the presence of a nontrivial condition on the branching measure. This is joint work with Adam Kinnison(Bath).

Peter Müller (Ludwigs Maximilians University Munich): Sa, 10:20-11:05
Ergodic properties of randomly coloured aperiodic point sets

We provide a framework for studying randomly coloured point sets over a suitable locally compact space. We first construct and describe an appropriate dynamical system for rather general uncoloured point sets and establish an ergodic theorem. This framework allows to incorporate a random colouring of the point sets. We derive an ergodic theorem for randomly coloured point sets and characterise ergodic measures. Special attention is paid to the exclusion of exceptional instances for uniquely ergodic systems. The setup allows for a straightforward application to randomly coloured graphs. This is joint work with Christoph Richard.

Sebastian Müller (Weizmann Institut, Israel): Sa, 14:20-14:40
Trace of branching random walk

We prove that the trace of BRW on Cayley graphs is recurrent for any BRW. Besides this we suggest a list of questions and conjectures that may lead to a better understanding of structural, spectral, and ergodic properties of the range and trace of BRW (and the underlying graph). (joint work with Itai Benjamini)

Tatiana Nagnibeda (Geneva University): Su, 11:00-11:45
Amenability and percolation

The aim of the talk will be to demonstrate the advantages of the study of percolation on arbitrary transitive graphs (not necessarily lattces in Eucledian spaces) from the viewpoints of both percolation theory and group theory. We shall concentrate in particular on the problem of various characterizations of amenable graphs and groups.

Erin Pearse (University of Iowa): Sa, 16:10-16:30
Resistance analysis of infinite networks

We consider a network (a connected graph with weighted edges) and functions u and v which are defined on its vertices and have finite Dirichlet energy E. The Laplacian L may be unbounded, and on transient networks, and the usual formula E(u,v)=¡u,Lv¿ extends to include a new (boundary) term. This extends the usual analogy to the Gauss-Green formula. We use this to construct a new boundary representation for the harmonic functions of finite energy, using Gel'fand triples and techniques from stochastic integration. A key tool is a reproducing kernel for the functions of finite energy; these elements are images of the vertices under natural embedding induced by the resistance metric.

Christoph Pittet: (University of Aix-Marseille 1) Sa, 9:30-10:15
Return probabilities and spectral distribution of Laplace operators

We estimate the spectral distribution of Laplace operators near zero with the help of isoperimetric inequalities (joint work with A. Bendikov, B. Bobikau, R. Sauer).

Jörg Schmeling (Lund University): Su, 11:50-12:35

Random trees generated by a dynamical system and the structure of typical orbits

A finite partition of a space $X = X_1 \cup \cdots X_k$ defines for each natural number n a tree of all words of length n. Given a dynamical system $f: X \to X$ we consider the subtree generated by a finite itinerary of an initial point $x \in X$, i.e. all words of length n in the itinerary of the point x up to time K. If we consider in addition an invariant measure μ the paths in the tree will be weighted according to the invariant measure. The question we study is what one can say about the dependence of the subtree on n, K and x. We will discuss the case when f is an expanding map of the circle and μ is an equilibrium state. it turns out that these random subtrees contain an "almost deterministic" and a "completely random" part. If K is choosen to be of the form $\exp(cn)$ several phase transition take place when we change c. Almost surely for large c the deterministic part is leading while for small c the random part constitutes most of the tree. Moreover in both cases a substantial amount of very rare events has to occur. Further the deterministic part is completely described by the thermodynamic formalism of the measure μ while the random part comes from combinatorial considerations. This analysis allows to describe the structure of a typical orbit and has applications to dyadic diophantine approximations on Cantor sets. The talk is based on joint work with Ai-Hua Fan and Serge Troubetzkoy.

Tatjana Schmeling-Turova (Lund University): Sa, 11:20-12:05

Asymptotic size of the largest cluster in inhomogeneous random graphs: sub-critical and critical phases

We study the sub-critical and critical phases of inhomogeneous random graphs. We derive an exact formula for the asymptotic size of the largest connected component normalized by $1/\log n$. In particular, we discover that the same well-known equation for the survival probability, whose positive solution determines the asymptotic of the size of the largest component in the super-critical case, plays the crucial role in the sub-critical case as well. But now these are the negative solutions which are the object of study.

At the critical phase we find a diffusion approximation for the normalized by $n^{-2/3}$ asymptotic component sizes of the graph.

Wolfgang Spitzer (Universität Erlangen): Sa, 14:40-15:00
Absolutely Continuous Spectrum for a Percolation Tree Model

We consider a binary tree model, where one and only one of the two forwarding neighbours is deleted with probability p. Then we prove that for small p, the spectrum of the Laplacian (or adjacency matrix) contains almost surely an interval of pure absolutely continuous spectrum. This is joint work with R. Froese and D. Hasler.

Ivan Veselić (Technische Universität Chemnitz): Su, 17:00-17:45
Percolation clusters on Calyey graphs and their spectra

We discuss geometric properties of percolation clusters of the lattice Z^d and of general Cayley graphs. Thereafter we turn to spectral properties of Laplacians on the full Cayley graph and on percolation sub-graphs. The considered properties are encoded in the spectral distribution function. In particular, we discuss approximability by finite volume eigenvalue counting functions, and the asymptotic behaviour at low energies.

Radoslaw Wojciechowski (Lissabon): Sa, 15:50-16:10
Stochastic Incompleteness of the Heat Kernel on Graphs

We consider the question of what geometric conditions imply the stochastic incompleteness (or explosion) of the diffusion process modeled by the heat kernel on infinite graphs.