Advanced and algorithmic graph theory

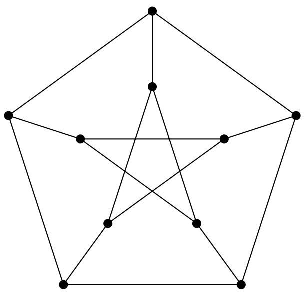
Summer term 2017

Exercise sheet 5

Exercises for the exercise session on 12/06/2017

Problem 5.1. Prove directly (that is, without using any results about edge-colourings from the lecture) that every k-regular bipartite graph is k-edge-colourable. Prove that this implies Theorem 4.11, i.e. $\chi'(G) = \Delta(G)$ for every bipartite graph.

Problem 5.2. Determine the chromatic index of the complete graph K^n and of the Petersen graph depicted below.



Problem 5.3. Prove Proposition 4.19 ($\mathrm{ch}(G) \leq 1 + \max_{H \subseteq G} \delta(H)$ for every G) and Theorem 4.20 (if G is neither complete nor an odd cycle, then $\mathrm{ch}(G) \leq \Delta(G)$).

Problem 5.4. For every $k \in \mathbb{N}$, find a bipartite graph G_k and an assignment of lists that shows that $\mathrm{ch}(G_k) \geq k$.

Problem 5.5. For positive integers r, s, we denote by K_s^r the graph with vertex set the disjoint union of sets $V_1, \ldots, V_r$ of size s and, for all $1 \leq i < j \leq r$, all edges between V_i and V_j. Prove that $\mathrm{ch}(K_2^r) = r$.

Hint. Try to use induction on r. If the induction step fails, what does this tell us about the lists?

Problem 5.6. A *total colouring* of G is a function $c\colon V \cup E \to S$ (with S the set of colours) such that $c|_V$ and $c|_E$ are vertex- and edge-colourings, respectively, and in addition no edge has the same colour as one of its end vertices. We write $\chi_t(G)$ for the least k for which there exists a total colouring of G with k colours.

Prove that the list colouring conjecture would imply $\Delta(G)+1 \leq \chi_t(G) \leq \Delta(G)+3$. (The *total colouring conjecture* asserts that even $\chi_t(G) \leq \Delta(G) + 2$.)