
Exercise sheet 4

Exercises for the exercise session on 2 May 2018

Problem 4.1. An *alignment* is a sequence of cycles. Let $A(z) = \sum_n A_n \frac{z^n}{n!}$ denote the exponential generating function for the class of alignments. Let X_n denote the number of cycles in a random alignment, which is chosen uniformly at random among all alignments of size n . Derive

- (1) a closed expression for $A(z)$;
- (2) an asymptotic expression for A_n ;
- (3) $\mathbb{E}(X_n)$.

Problem 4.2. Let Y_n denote the number of components in a random 2-regular graph, which is chosen uniformly at random among all 2-regular (labelled simple) graph with vertex set $[n]$. Derive

- (1) $\mathbb{E}(Y_n)$
- (2) $\mathbb{V}(Y_n)$

Problem 4.3. Let $B(z) = \sum_n B_n z^n$ denote the ordinary generating function for the class of binary strings with no consecutive 0's (note: the empty string is included in this class). Derive

- (1) a closed expression for $B(z)$;
- (2) an asymptotic expression for B_n .

Problem 4.4. Let $Q(z) = \sum_n Q_n \frac{z^n}{n!}$ denote the exponential generating function for the class of permutations with cycles of length $2 \pmod{3}$. Derive

- (1) a closed expression for $Q(z)$;
- (2) an asymptotic expression for Q_n .