Exercise 12

Let X and Y be the values of two independently thrown six-sided dice. Calculate

(a) $H(X) + H(Y)$,

(b) $H(X + Y)$,

(c) $H(X - Y)$,

(d) $H(\max(X, Y))$.

Exercise 13

A fair coin is flipped until head occurs the first time. Let X denote the number of flips required.

(a) Find the entropy $H(X)$ in bits.

(b) Find an efficient sequence of yes-no questions in order to guess the outcome of X. Compare $H(X)$ to the expected number of questions.

Exercise 14

The play-offs of NBA are played between team A and team B in a Best-of-Seven series that terminates as soon as one of the teams wins four games. Let the random variable X represent the outcome of the series of games (some possible values are AAAA, BABABAB and AAABBBB). Let Y denote the number of games played. Assuming that the two teams are equally strong, determine the values of $H(X)$, $H(Y)$, $H(Y \mid X)$ and $H(X \mid Y)$.

Exercise 15

Let X and Y be random variables that take values in finite sets $\mathcal{X}$ and $\mathcal{Y}$, respectively. Let $Z = X + Y$.

(a) Show that $H(Z \mid X) = H(Y \mid X)$. Argue that if X, Y are independent, then $H(Y) \leq H(Z)$ and $H(X) \leq H(Z)$.

(b) Give an example of (necessarily dependent) random variables in which $H(X) > H(Z)$ and $H(Y) > H(Z)$.

(c) Find necessary and sufficient conditions for $H(Z) = H(X) + H(Y)$.